Introduction to Matching and Allocation Problems (II)

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Organization of This Lecture

- (Review of) One-to-One "Marriage" Matching
- Many-to-One "College Admissions" Matching
- (Brief Comments on) Many-to-Many Matching
- Many-to-One Matching with Transfers

The Marriage Problem

Question

In a society with a set of men M and a set of women W, how can we arrange marriages so that no agent wishes for a divorce?

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Assumptions

1. Agents have strict preferences(!).
2. Bilateral relationships: only pairs (and possibly singles).
3. Two-sided: men only desire women; women only desire men.
4. Preferences are fully known.

The Deferred Acceptance Algorithm

Step 1

1. Each man "proposes" to his first-choice woman.
2. Each woman holds onto her most-preferred acceptable proposal (if any) and rejects all others.

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Step $t \geq 2$

1. Each rejected man "proposes" to the his favorite woman who has not rejected him.
2. Each woman holds onto her most-preferred acceptable proposal (if any) and rejects all others.

At termination, no agent wants a divorce!

Stability

Definition

A **matching** μ is a one-to-one correspondence on $M \cup W$ such that

- $\mu(m) \in W \cup \{m\}$ for each $m \in M$,
- $\mu(w) \in M \cup \{w\}$ for each $w \in W$, and
- $\mu^2(i) = i$ for all $i \in M \cup W$.

Definition

A marriage matching μ is **stable** if no agent wants a divorce.

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Definition

A marriage matching μ is **stable** if no agent wants a divorce:

- **Individually Rational**: All agents i find their matches $\mu(i)$ acceptable.
- **Unblocked**: There do not exist m, w such that both

$$m \succ_w \mu(w) \quad \text{and} \quad w \succ_m \mu(m).$$

Existence and Lattice Structure

Theorem (Gale–Shapley, 1962)

A stable marriage matching exists.

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A stable marriage matching exists.

Theorem (Conway, 1976; Knuth, 1976)

- *Given two stable matchings* μ, ν, *there is a stable match* $\mu \vee \nu$ ($\mu \wedge \nu$) *which every man likes weakly more (less) than* μ *and* ν.
- *If all men (weakly) prefer stable match* μ *to stable match* ν, *then all women (weakly) prefer* ν *to* μ.
- *The man- and woman-proposing deferred acceptance algorithms respectively find the man- and woman-optimal stable matches.*

(Two-Sidedness is Important)

Consider four potential roommates:

$$P^1 : 2 \succ 3 \succ 4 \succ \emptyset,$$
$$P^2 : 3 \succ 1 \succ 4 \succ \emptyset,$$
$$P^3 : 1 \succ 2 \succ 4 \succ \emptyset,$$
$$P^4 : \text{w/e}.$$

$\rightsquigarrow$ No stable roommate matching exists!

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(But wait until Wednesday....)

Opposition of Interests: A Simple Example

$$\succ_{m_1} : w_1 \succ w_2 \succ \emptyset \qquad \succ_{w_1} : m_2 \succ m_1 \succ \emptyset$$

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- This opposition of interests result also implies that there is no mechanism which is **strategy-proof** for both men and women.

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Weak Pareto Optimality

Theorem (Roth, 1982)

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$\Rightarrow$ Any woman who gets a last-stage proposal in deferred acceptance has not "held" any men.

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$\Rightarrow$ Any woman who gets a last-stage proposal in deferred acceptance has not "held" any men.

$\Rightarrow$ At least one woman in $\bar{\mu}(M)$ is single under μ $\Rightarrow\Leftarrow$.

Incentives

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No stable matching mechanism exists for which stating true preferences is a dominant strategy for every agent.

Theorem (Dubins–Freedman, 1981; Roth, 1982)

The male-optimal stable matching mechanism makes it a dominant strategy for each man to state his true preferences.

The College Admissions Problem (I)

Question

In a society with a set of students S and a set of colleges C, how can we assign students to colleges in a stable fashion?

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In a society with a set of students S and a set of colleges C, how can we assign students to colleges in a stable fashion?

Assumptions

1. Agents have strict preferences(!).
2. Students have unit demand.
3. Schools have *responsive* preferences (defined on the next slide).
4. Two-sided; preferences are fully known.

The College Admissions Problem (II)

Definition

The preferences P^c of college c *over sets of students* are **responsive** if they are consistent with

1. a complete, transitive preference relation $\succ_c$ over students and
2. a quota q_c.

That is, for all $S' \subseteq S$ with $|S'| < q_c$, and any students $i, j \in S \setminus S'$,

1. $(S' \cup \{i\}) P^c (S' \cup \{j\}) \iff i \succ_c j$.
2. $(S' \cup \{i\}) P^c S' \iff i \succ_c \emptyset$.

The College Admissions Problem (III)

Definition

A **matching** μ is a correspondence on $S \cup C$ such that

- $\mu(s) \in C \cup \{s\}$ for each $s \in S$,
- $\mu(c) \subseteq S$ for each $c \in C$, and
- $s \in \mu(\mu(s))$ for all $s \in S$.

Stability

Definition

A matching μ is **(pairwise) stable** if:

- **Individually Rational**: All agents i find their matches $\mu(i)$ acceptable.
- **Unblocked**: There do not exist s, c such that $c \succ_s \mu(s)$ and

$$s \succ_c s' \text{ for some } s' \in \mu(c) \quad \text{or} \quad s \succ_c \emptyset \text{ and } |\mu(c)| < q_c.$$

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N.B. When college preferences are responsive (indeed, when they are *substitutable*), pairwise stability is equivalent to group stability and being in the core.

A Related One-to-One Market...

- Replace each college c with $c_1, \ldots, c_{q_c}$.

- Modify students' preferences:

$$c' \succ_s c \succ_s c'' \quad \Longrightarrow \quad c' \succ_s c_1 \succ_s \cdots \succ_s c_{q_c} \succ_s c''$$

Theorem (Roth–Sotomayor, 1990)

A college admissions matching is stable if and only if the corresponding matching in the related one-to-one market is stable.

$\Rightarrow$ *A stable college admissions matching exists!*

Existence and Lattice Structure

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A stable college admissions matching exists.

Theorem

- *Given two stable matchings μ, ν, there is a stable $\mu \vee \nu$ ($\mu \wedge \nu$) which every college likes weakly more (less) than μ and ν.*
- *If all colleges (weakly) prefer stable match μ to stable match ν, then all students (weakly) prefer ν to μ.*
- *There exist college- and student-optimal stable matchings (and we can find them via deferred acceptance!).*

The "Rural Hospitals" Theorem

Theorem (Roth, 1986)

At every stable matching

1. *the same students are matched, and*
2. *the same college positions are filled.*

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Proof

- Use the Lone Wolf Theorem in the related one-to-one market. . . .

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Proof

- Use the Lone Wolf Theorem in the related one-to-one market. . . .
- Look at a college c that does not fill all its positions at the college-optimal stable matching $\bar{\mu}$; consider some other stable matching μ; and suppose that $\bar{\mu}(c) \neq \mu(c)$. . . .

Incentives (I)

Theorem (Roth, 1985)

The student-optimal stable matching mechanism makes it a dominant strategy for each student to state his true preferences.

However, no other stable matching mechanism makes it a dominant strategy for each student to state his true preferences.

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Proof

- Use the incentives theorems in the related one-to-one market. . . .

Incentives (III)

Theorem

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Proof

- Lattice structure + truncation theorem. . . .

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"Dropping" Strategy: Consider a market with three colleges and four students, with $q_{c_1} = 2$ and $q_{c_2} = q_{c_3} = 1$.

$$\succ_{s_1} : c_3 \succ c_1 \succ c_2 \succ \emptyset \qquad \succ_{c_1} : s_1 \succ s_2 \succ s_3 \succ s_4 \succ \emptyset$$
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$$\succ_{s_4} : c_1 \succ c_2 \succ c_3 \succ \emptyset$$

Unique stable matching: c_1–$\{s_3, s_4\}$; c_2–$\{s_2\}$; c_3–$\{s_1\}$.

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Unique stable matching: c_1–$\{s_3, s_4\}$; c_2–$\{s_2\}$; c_3–$\{s_1\}$.
If c_1 "drops" s_2 and s_3: c_1–$\{s_1, s_4\}$; c_2–$\{s_2\}$; c_3–$\{s_3\}$.

Substitutable Preferences

Definition

The preferences of college c are **substitutable** if for all $i, j \in S$ and $S' \subseteq S$, if $i \notin C^c(S' \cup \{i\})$, then $i \notin C^c(S' \cup \{i, j\})$.

i.e. There is no student j that (sometimes) "complements" i, in the sense that gaining access to i makes j more attractive.

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i.e. There is no student j that (sometimes) "complements" i, in the sense that gaining access to i makes j more attractive.

- Key results for responsive preferences (e.g., the existence of stable matchings) generalize to the case of substitutable preferences. (More on this on Wednesday. . . .)
- However, the "related one-to-one market" construction does not work, so we need direct arguments(!).

Weak Pareto Optimality

Theorem (Kojima, 2008)

The student-optimal stable matching is weakly Pareto optimal for students "if and only if" the preferences of every college are substitutable and satisfy the law of aggregate demand.[a]

[a] That is, $|C^c(S'')| \leq |C^c(S')|$ whenever $S'' \subseteq S' \subseteq S$.

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- Additionally, Romm (forth.) proves welfare comparative statics in the case that the preferences of every college are substitutable and satisfy the law of aggregate demand(!).

Remarks on Many-to-Many Matching

- Many-to-**MANY** Definitions of Stability. . . .
  see Sotomayor (1999); Echenique and Oviedo (2006); Konishi and Ünver (2006); . . .
- Pairwise Stable $\ncong$ Core.
  see Blair (1988)
- Pairwise Stable $\cong$ Stable only when preferences are substitutable.
- Nevertheless, key existence and structural results hold in the presence of substitutable preferences.

Kelso–Crawford (1982)

Main Results

In two-sided, many-to-one matching markets with

1. *bilateral contracts,*
2. *transferable utility, and*
3. *substitutable preferences,*

competitive equilibria exist and coincide with $\{stable, core\}$ *outcomes.*

The Setting

- m workers, n firms; many-to-one matching
- Workers care about wages and employers, but not colleagues.
- Firms care about their wages and employees.

The (Gross) Substitutability Condition

Definition

Workers are **(gross) substitutes** for j if for any two salary vectors s_j and s'_j with $s_j \leq s'_j$, for each $Y \in D_j(s_j)$, there is some $Y' \in D_j(s'_j)$ such that

$$\{i \in Y : s_{ij} = s'_{ij}\} \subseteq Y'.$$

The Salary Adjustment Process (I)

1. Firms face a set of salaries.
2. Firms make offers to their most preferred set of workers. Any previous offer that was not rejected must be honored.
3. Workers evaluate offers and tentatively hold their best acceptable offers.
4. For each rejected offer, increment the feasible salary for the rejecting worker–firm pair.
5. If no new offers are made, terminate the process and implement the outcome; otherwise, iterate.

The Salary Adjustment Process (II)

Theorem

1. *The adjustment process terminates.*
2. *The final allocation is (generically) unique.*
3. *The final outcome is*
    1. *in the core, and*
    2. *firm-optimal.*

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- Sound familiar?
- Discrete vs. continuous adjustment?
- Necessity of substitutability?

Similarities... and differences!

- One-to-One “Marriage” Matching
- Many-to-One “College Admissions” Matching
- Many-to-Many Matching
- Many-to-One Matching with Transfers