Concordance among Holdouts

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(joint work with E. Glen Weyl, Harvard Society of Fellows)

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The Holdout Problem: A Simple Example

- Ten farmers own (privately valued) farms

You want to buy the farms and build an airfield (worth 90). All you know is that farmers' values are uniformly drawn from \( \{1, \ldots, 10\} \) (expected total value 55). What should you do?

- Take-it-or-leave-it offers of 1, \ldots, 10 (total 55)?
- Take-it-or-leave-it offers of 8 (total 80)?
- Self-assessment: ask each farmer to reveal his value?
- Eminent domain: take land and pay each farmer 1 (total 10)?
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Kominers and Weyl (2010)
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The Holdout Problem

- Holdout is pervasive.
  - Perfect complements problems
    - land assembly, corporate acquisitions, spectrum recovery
  - All trade dries up as $N \to \infty$.

- Institutions for reducing holdout are primitive.
  - Takings; voting-based procedures

- Sharp contrast to the case of auctions for substitutes, where even naïve designs are efficient as $N \to \infty$ (Bulow & Klemperer (1996))
Our Contributions

1. Introduce holdout as a market design problem
   - Design goals
     - straightforwardness, bilateral efficiency, partial property rights

2. Propose a class of solutions
   - Design principle — “Concordance” — which ensures key goals
   - Concordance mechanisms: a market design for holdout
Road Map

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2. Road Map (← we are here)
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   - Market Design Goals
   - Applications
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   - Straightforward Concordance
   - Other Concordance Mechanisms
   - X-plurality
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   - X-plurality
6. Conclusion
Basic Model (in language of land assembly)

- Buyer has (private) value $b$ for aggregate plot.
- Submits offer $o$ (recommended $o^*$).
- Each seller $i$ has (private) value $v_i$ for her subplot. Reports reserve value $r_i$ (recommended $r_i^*$).
- Each seller has expected share of total value $s_i$.
  - Can be entirely exogenous or determined by buyer.
  - $s_i$ close to $v_i / \left( \sum_j v_j \right) \Rightarrow$ better property rights.

A mechanism is a transaction procedure.
Basic Model (in language of land assembly)

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- A *mechanism* is a transaction procedure
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\[
\begin{array}{cccccccccc}
V_1 & V_2 & V_3 & V_4 & V_5 & V_6 & V_7 & V_8 & V_9 & V_{10} \\
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  - Perfectly observed \(s_i = \frac{v_i}{(\sum_j v_j)}\)?
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Design Goals: The Ideal

1. Fully Efficient: mechanism captures all gains from trade
   - Sale $\iff b \geq \sum_i v_i \equiv V$

2. Protects Individual Property Rights: no seller sells below value
   - Sale $\implies$ each seller $i$ receives at least $v_i$

3. Budget-balanced
   - No transfers to/from the market-maker
Design Goals: Our Proposal

1. Straightforward for Sellers: truthful play dominant
   - \( r^*(v_i) = v_i \); dominant-strategy equilibrium

2. Bilaterally Efficient: as efficient as bilateral trade
   - Sale \( \iff o^*(b) \geq V \)

3. Protects Partial Property Rights
   - Collective PR: community not forced to sell for less than \( V \)
   - Approximate Individual PR: seller \( i \) receives at least \( \frac{s_i(V-v_i)}{1-s_i} \)

4. Self-financing
   - No transfers from the market-maker
Examples of Holdout

1. Land assembly
   - Eminent domain/takings
     - Government assesses and pays compensation (⇒ corruption)
     - But relative valuations reasonable to measure? (⇒ shares)
   - Collective ownership (e.g. *ejido*)
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2. Corporate acquisitions
   - To protect minority shareholders, credible full offer required
   - Shares explicit; Voting rules standard for decision
   - Collective property rights protect collective investments
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2 Corporate acquisitions
   - To protect minority shareholders, credible full offer required
   - Shares explicit; Voting rules standard for decision
   - Collective property rights protect collective investments

3 Other examples
   - Debt settlements; Spectrum reassembly; Multi-plaintiff lawsuits;
     Patent pools; Art collections
Very few commodities are consumed in just the form in which they are left in the hands of the first producer. . .

Several raw materials are generally brought together in the manufacture of each of these products. . .

The more there are of articles thus related, the higher the price determined by the division of monopolies will be, than that which would result from the fusion or association of monopolists.

—Cournot (1838)
The Concordance Principle

- Cournot’s Two-part Solution
  1. Sellers merge and divide revenues
  2. Each seller internalizes others’ profits/losses
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- Concordance Principle is analogous
  1. Sellers divide offer into previously-specified shares
  2. Each seller pays a pigouvian tax for externalities

Formally: A mechanism satisfies the Concordance Principle if

\[ o \geq R \equiv \sum_i r_i \]

\[ 1 - s_i = \Rightarrow \text{Noninfluential sellers}\]
\{\text{pay no tax, get at least } s_i \text{ in sale}\}

\[ r^*(v) = v; o^*(b) \text{ is monopsonist-optimal offer} \]
The Concordance Principle

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  1. Sellers merge and divide revenues
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- Formally: A mechanism satisfies the Concordance Principle if
  0. Offer accepted when \( o \geq R \equiv \sum_i r_i \)
  1. \( r_i = s_i o \) is “no influence”
     \[ \implies \text{Sale } \iff o \geq R_i \equiv \frac{\sum_{j \neq i} r_j}{1-s_i} \]
     \[ \implies \text{Noninfluential sellers } \{ \text{pay no tax, get at least } s_i o \text{ in sale} \} \]
  2. Influential sellers may pay a tax to encourage truthfulness
     \( r^*(v) = v; o^*(b) \) is monopsonist-optimal offer
Mechanism Design

Concordance principle

+ Auction enforcement

Concordance mechanism
The Simple Example Revisited Once More

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- You want to buy the farms and build an airfield (worth $b = 90$)
  - Offer $o = 56 < o^*(b)$

- Straightforward Concordance (Externality Tax)
  - Shares perfectly observed $\implies$ no taxes ($\implies$ full PRs)
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    - Sale occurs; Farmer 10 is pivotal (\( R_{10} \approx 56.84 \)) and is taxed his externality (\( \tau_{10} = (1 - \frac{1}{20})|56.84 - 56| \approx .8 \)).
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Properties of Concordance Mechanisms

Theorem

Concordance mechanisms are bilaterally efficient, and are fully efficient as $N \to \infty$. 

Kominers and Weyl (2010)
Properties of Concordance Mechanisms

Theorem

Concordance mechanisms are bilaterally efficient, and are fully efficient as $N \to \infty$.

Proof

- Sellers report truthfully; buyer gives monopsonist-optimal offer
- Outcome same as bilateral bargain (Myerson-Satterwaite (1981)) between buyer and single seller with value $V$
- Uncertainty about $V = \sum_i v_i$ vanishes as $N \to \infty$
Properties of Concordance Mechanisms

**Theorem**

*Concordance mechanisms are bilaterally efficient, and are fully efficient as* \( N \to \infty \).

**Theorem**

*Concordance mechanisms preserve collective and approximate individual property rights.*
Straightforward Concordance (SC)

Concordance + Vickrey-Clarke-Groves + Cavallo (2006)

1. Straightforward for sellers (VCG proof)
2. Self-financing (refund is designed this way)
3. Implementable (buyers recommended optimal offer)
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- The refund we choose is maximal among self-financing, nondiscriminatory mechanisms.
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Straightforward Concordance is unique/optimal in the sense that

- Any truthful Concordance mechanism is VCG with refund.
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Still, Straightforward Concordance has some problems:

- Imperfect budget-balance; collusion
- Monetary payments, risk and individual budgets
Bayes-Nash Concordance (BNC)

- Expected Externality mechanism $\implies$ Bayes-Nash implementable
- Budget-balanced; Strictly preserves collective property rights
- Less risky for sellers; less collusive(?)
Other Concordance Mechanisms

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2. All-pay Concordance (APC)
   - Retains benefits of BNC over SC but not truthful $\implies$
     Equilibrium behavior unclear
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4. Other possibilities: core-nearest, other package auction rules
X-plurality

Voting on sale (given shares)

1. Sale occurs \( \iff \) \( X \)% of shares favor sale
2. If sale, each seller \( i \) receives \( s_i o \)
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1. Sale occurs $\iff X\%$ of shares favor sale
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- Encompasses all holdout mechanisms used before
  - $X = 0 \sim$ eminent domain: pay market value (minimum)
  - $X$ midrange $\sim$ corporate acquisitions; Heller and Hills (2008)
  - $X$ high $\sim$ decentralized bargaining; Shapiro and Pincus (2007)
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  - $X$ high $\sim$ decentralized bargaining; Shapiro and Pincus (2007)
- Simple, balanced, straightforward, no extra money/risk
- Protects $X$ percent of property rights
- $X$ must match with distribution of values
- Raises many issues
  - Share-weighting, right $X$, small population, trade distortion
## Comparing Mechanisms

<table>
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<tr>
<th>Mechanism</th>
<th>Finances</th>
<th>Simplicity</th>
<th>Efficiency</th>
<th>Property Rights</th>
<th>Risk and Budgets</th>
<th>Share incentive</th>
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<td>SC</td>
<td>Self-financing, asymptotically balanced</td>
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<td>Collective, approx.</td>
<td>High</td>
<td>Yes</td>
<td>Moderate?</td>
<td></td>
</tr>
<tr>
<td>BNC</td>
<td>Balanced budget</td>
<td>Implementable</td>
<td>Bilateral, asymptotic</td>
<td>Strict collective, approximate individual</td>
<td>Low</td>
<td>Yes</td>
<td>Low?</td>
<td></td>
</tr>
<tr>
<td>APC</td>
<td>Balanced budget</td>
<td>Approx. implementable with small sellers?</td>
<td>Bilateral, asymptotic</td>
<td>Same as BNC</td>
<td>Low</td>
<td>Yes</td>
<td>None?</td>
<td></td>
</tr>
<tr>
<td>FPC</td>
<td>Balanced budget</td>
<td>Very complex, likely unimplementable</td>
<td>Bilateral, asymptotic</td>
<td>Same as BNC</td>
<td>Moderate</td>
<td>Yes</td>
<td>Very low?</td>
<td></td>
</tr>
<tr>
<td>X-plurality (low X)</td>
<td>Budget balanced</td>
<td>Like SC</td>
<td>Too many sales</td>
<td>None</td>
<td>None</td>
<td>Yes</td>
<td>None</td>
<td></td>
</tr>
<tr>
<td>X-plurality (mid X)</td>
<td>Budget balanced</td>
<td>Like SC</td>
<td>If percentile matches mean</td>
<td>X of shares, approximate individual if efficient</td>
<td>None</td>
<td>No</td>
<td>High?</td>
<td></td>
</tr>
<tr>
<td>X-plurality (high X)</td>
<td>Budget balanced</td>
<td>Like SC</td>
<td>Holdout: no approx. gains</td>
<td>Near-perfect individual</td>
<td>None</td>
<td>Yes</td>
<td>Very high?</td>
<td></td>
</tr>
</tbody>
</table>
Recap

1. We introduced holdout as a market design problem
   - Achievable design goals
     - straightforwardness, bilateral efficiency, partial property rights

2. We proposed a class of solutions
   - Concordance principle and associated mechanisms
Future Directions
Future Directions

1. Analytic extensions
   - Implementing BNC
   - Optimal $X$ for $X$-plurality
   - Measuring losses to holdout

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Implementing BNC
Optimal $X$ for $X$-plurality
Measuring losses to holdout
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   - Optimal X for X-plurality
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2. Improving the mechanisms
   - Partial property rights
   - Limited, privately-known budgets (Pai and Vohra (2009))
Future Directions

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   - Optimal $X$ for $X$-plurality
   - Measuring losses to holdout

2. Improving the mechanisms
   - Partial property rights
   - Limited, privately-known budgets (Pai and Vohra (2009))

3. Broader directions
   - Other Concordance mechanisms
   - Non-Concordance solutions, other PRs
   - Imperfect complements; competing groups
     1. Price theory analysis
     2. Mechanism design analysis
     3. Practical solutions/extensions
But wait...

Isn’t “holdout” ~ strategically lying to demand more surplus?
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- Shapiro and Pincus (2007) propose solution
  1. Each seller is assigned a “share” (probably by buyer)
  2. Buyer makes an offer, with sale if all sellers accept
- No incentive for sellers to lie...
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Holdout is a fundamental of complements design

- not just a strategic problem

To solve holdout, we must solve the basic problem(!)
The law [in preindustrial France] granted every owner of grazing rights a veto over the enclosure. Compensating the owners for their grazing rights—one solution suggested by that bit of economics known as the Coase theorem—was impractical. It would be difficult to specify what the grazing rights were worth, and each owner had reason to exaggerate their value. Each one, indeed, could hold out and threaten to block the enclosure in the hope of gaining a share of the farmers gains. The veto, in short, transformed the owners of grazing rights into monopolists and left the farmer at their mercy. The price he would need to pay for their consent could easily make artificial meadows a losing proposition. (Hoffman (1988))
Historical Holdout

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In England enclosures [...] had faced the hurdle of unanimity until private acts of Parliament let owners of four-fifths of the land override minority opposition. Common by the 1760s, the English procedure greatly reduced bargaining costs and facilitated both enclosure and more general improvements. (Hoffman (1988))
Property Rights

Theorem

*Concordance mechanisms preserve collective and approximate individual PRs.*
Property Rights

Theorem

Concordance mechanisms preserve collective and approximate individual PRs.

Proof

- Sellers report \( s_i V \implies \text{sale} \iff o \geq R = \sum_i s_i V = V \)

- If seller \( i \) reports \( r_i = s_i o \) (indifference), then
  - Sale \( \iff o \geq R_i = \frac{\sum_{j \neq i} r_j}{1-s_i} \)
  - Seller \( i \) receives at least \( s_i R_i = s_i \frac{\sum_{j \neq i} v_j}{1-s_i} = \frac{s_i(V-v_i)}{1-s_i} \)

Kominers and Weyl (2010)
Efficiency

Theorem

Concordance mechanisms are fully efficient as $n \to \infty$. 

Proof

There exists an $M > 0$ such that $n s_n < M$ for all $n, i$.

1. $v_n$ is $n_i$ are i.i.d. across $n$ and $i$ from some distribution with finite support and $b$ is drawn i.i.d. across $n$.

2. $E[v_n] = \mu$; $V[v_n] < M^2 \sigma^2$ $\Rightarrow$ $p[v_n - \mu \geq \alpha] \leq \frac{M^2 \sigma^2 + n \alpha^2}{\mu^2} \to 0$ probability of sale $\arg\max q q(q(s_n)) \equiv \tilde{q}_n(b) \to 1$ inefficiency $\int_\infty \mu(1 - \tilde{q}_n(b))(b - \mu) h(b) db \to 0$

analogous argument when $b < \mu$
Efficiency

Theorem

Concordance mechanisms are fully efficient as $n \to \infty$ if

1. There exists an $M > 0$ such that $ns_i^n < M$ for all $n, i$.
2. $\left\{ \frac{v_i^n}{s_i^n} \right\}_{i=1}^n$ are i.i.d. across $n$ and $i$ from some distribution with finite support and $b$ is drawn i.i.d. across $n$. 

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### Proof

- $E[V^n] = \mu; \quad V^n < \frac{M^2\sigma^2}{n} \quad \implies \quad p[V^n - \mu \geq \alpha] \leq \frac{M^2\sigma^2}{M^2\sigma^2 + n\alpha^2} \to 0$
- probability of sale $\arg\max_q q(b - S_n(q)) \equiv \tilde{q}_n(b) \to 1$
- inefficiency $\int_{\mu}^{\infty} (1 - \tilde{q}_n(b))(b - \mu)h(b) \, db \to 0$
- analogous argument when $b < \mu$
Straightforward Concordance (SC)

Simplest approach: Vickrey-Clarke-Groves

1. If pivotal in sale decision, pay Pigouvian tax of 
   \[(1 - s_i)|R_i - o|\]

2. Receive refund of 
   \[s_i \min_{\hat{r}_i} \sum_{j=1}^{N} \left( \frac{1}{\hat{R}_j - o}(\hat{R}_j - o)(1 - s_j)|o - \hat{R}_j| \right)\]

3. Rest follows from Concordance principle
Bayes-Nash Concordance (BNC)

Expected Externality

1. Pay tax of \((1 - s_i)E_{v-i} \left[ |V_i - o| 1_{(V_i-o)(V-o)<0} \mid v_i = r_i \right]

2. Receive refund of \(s_i \sum_{j \neq i} E_{v-j} \left[ |V_j - o| 1_{(V_j-o)(V-o)<0} \mid v_j = r_j \right]

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- Not straightforward but implementable and
  1. Budget-balanced
  2. Strictly preserves collective property rights
  3. Less risky for sellers; less collusive(?)

Violates Wilson doctrine(!)

Incentive properties depend on risk preferences

Kominers and Weyl (2010)
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- Violates Wilson doctrine(!)
- Incentive properties depend on risk preferences
All-pay Concordance (APC)

1. Pay tax of $|s_j o - r_j|$

2. Receive refund of

$$s_i \sum_{j \neq i} \frac{|s_j o - r_j|}{1 - s_j}$$

3. Rest follows from Concordance principle
Concordance Among Holdouts
Extra Slides

All-pay Concordance (APC)

1. Pay tax of $|s_jo - r_j|$

2. Receive refund of

$$s_i \sum_{j \neq i} \frac{|s_jo - r_j|}{1 - s_j}$$

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- Equivalently: Choose direction; Put up money; Biggest pool wins
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- Equivalently: Choose direction; Put up money; Biggest pool wins
- Retains benefits of BNC over SC but...
  - Truthfulness not incentive compatible
  - Equilibrium behavior unclear
- Revenue Equivalence?
All-pay Concordance (APC)

1. Pay tax of $|s_j o - r_j|$
2. Receive refund of
   \[ s_i \sum_{j \neq i} \frac{|s_j o - r_j|}{1 - s_j} \]
3. Rest follows from Concordance principle
   - Equivalently: Choose direction; Put up money; Biggest pool wins
   - Retains benefits of BNC over SC but...
     - Truthfulness not incentive compatible
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   - Revenue Equivalence?
   - BNC $\sim$ pay $f(v_i - s_i o)$ with $f(0) = 0$, $f'(x)x > 0$
   - Problem how to calculate $f$; could just plug in $|x|$
First-price Concordance (FPC)

1. Pay tax of $\max(0, [s_i o - r_i] 1_{\text{sale}}, [r_i - s_i o] 1_{\text{no sale}})$

2. Receive refund of

$$s_i \sum_{j \neq i} \frac{\max ([s_j o - r_j] 1_{\text{sale}}, [r_j - s_j o] 1_{\text{no sale}})}{1 - s_j}$$

3. Rest follows from Concordance principle
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\[
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\]

3. Rest follows from Concordance principle

- Once again...
  - Truthfulness not incentive compatible
  - Equilibrium behavior unclear

- Other possibilities: core-nearest, other package auction rules

Kominers and Weyl (2010)
Public Goods

Holdout problem \sim Closely related to public goods

- Good benefits everyone
- Switch signs for binary, quasi-linear public goods
- Voluntary \sim property rights; Lindahl pricing \sim perfect shares
  - People pay “tax” based on approximation to their shares
  - Quantity provided determined by demand at true shares
- That literature never found general implementation—why?
  - Focus very general: income, shapes, heterogeneity
  - Not very “practical” because no focus on applications
  - Voluntary participation focus
  - Approximations only natural in special case

(Also equivalent to original Cournot collaboration)
Other Proposals for Solving Holdout

1. Weighted Majority Voting
   - Heller and Hills (2008)

2. Property Self-assessment
   - Bell and Parchomovsky (2007)
   - Plassmann and Tideman (2009)

3. Secret Purchases
   - Kelly (2006)

4. Graduated Density Zoning
   - Shoup (2008)
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