Using Matching with Preferences over Colleagues to Solve Classical Matching Problems

Scott Duke Kominers

Harvard University

Boston Undergraduate Research Symposium April 11, 2009

April 11, 2009

1 / 13

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Image: Image:

College Admissions

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April 11, 2009 2 / 13

College Admissions

• Students

College Admissions

• Students, with preferences over colleges

- Students, with preferences over colleges
- Colleges

- Students, with preferences over colleges
- Colleges, with preferences over students

- Students, with strict preferences over colleges
- Colleges, with strict preferences over students

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College Admissions

- Students, with strict preferences over colleges
- Colleges, with strict preferences over students

Question

How do we match students to colleges?

College Admissions

- Students, with strict preferences over colleges
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Question

How do we match students to colleges in a stable way?

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What is "stability"?

Question How do we match students to colleges in a stable way?

What is "instability"?

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An matching of students to colleges is "unstable" if...
student s₁ matched to college Y

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An matching of students to colleges is "unstable" if...

• student s_1 matched to college Y $(s_1 \rightarrow Y)$

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How do we match students to colleges in a stable way?

- student s_1 matched to college Y $(s_1 \rightarrow Y)$
- student s_2 matched to college Z

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- student s_1 prefers college Z to college Y

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- student s_1 matched to college Y $(s_1 \rightarrow Y)$
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- student s_1 prefers college Z to college Y $(Z \succ_{s_1} Y)$
- college Z prefers student s_1 to student s_2

Question

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- student s_1 matched to college Y $(s_1 \rightarrow Y)$
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Why is instability bad?

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Good news: a stable matching always exists.

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Why is instability bad?

<u>Good news: a stable matching always exists.¹</u>

¹Gale–Shapley (1962)

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An Example

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Image: A mathematical states and a mathem

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Possible Matchings

• $s_1 \rightarrow Z$, $s_2, s_3 \rightarrow \emptyset$ — unstable

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Using Matching with Preferences over Colleagues

Real-world Applications

Scott Duke Kominers (Harvard)

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Matching of...

Matching of...

students to schools

Matching of...

• students to schools (in Boston and New York)

Matching of...

- students to schools (in Boston and New York)
- (medical) students to residencies

Matching of...

- students to schools (in Boston and New York)
- (medical) students to residencies
- students to sororities

Real-world Applications

Matching of...

- students to schools (in Boston and New York)
- (medical) students to residencies
- students to sororities

However...

Real-world Applications

Matching of...

- students to schools (in Boston and New York)
- (medical) students to residencies
- students to sororities

However...

• no direct application to college admissions

Real-world Applications

Matching of...

- students to schools (in Boston and New York)
- (medical) students to residencies
- students to sororities

However...

• no direct application to college admissions (yet)



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• We just described "classical matching".

- We just described "classical matching".
- Recall the title slide....

Using Matching with Preferences over Colleagues to Solve Classical Matching Problems

Scott Duke Kominers

Harvard University

Boston Undergraduate Research Symposium April 11, 2009

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Natural Question

- We just described "classical matching".
- Recall the title slide....

Natural Question

What is "matching with preferences over colleagues"?

The Problem

College Admissions

- Students, with strict preferences over colleges
- Colleges, with strict preferences over students

Question

College Admissions

- Students, with strict preferences over colleges
- Colleges, with strict preferences over students

Question

College Admissions

- Students, with strict preferences over colleges and over their possible sets of classmates
- Colleges, with strict preferences over students

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Question — Solved

College Admissions

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Question — Solved, with an Algorithm How do we match students to colleges in a stable way?

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^aEchenique-Yenmez (2007)

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Nontrivial Question

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College Admissions

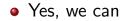
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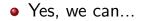
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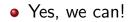
Nontrivial Question

- Yes, we can...
 - with an elementary construction...

Nontrivial Question

- Yes, we can...
 - with an elementary construction...
 - but at a complexity cost.

Nontrivial Question



Nontrivial Question

Can we use this algorithm to solve classical matching?

• Yes, we can!

Theorem

Nontrivial Question

Can we use this algorithm to solve classical matching?

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Theorem For any "classical matching" problem

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Can we use this algorithm to solve classical matching?

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Theorem

For any "classical matching" problem, there is an associated "matching with preferences over colleagues" problem

Nontrivial Question

Can we use this algorithm to solve classical matching?

• Yes, we can!

Theorem

For any "classical matching" problem, there is an associated "matching with preferences over colleagues" problem with stable matchings directly corresponding to the stable matchings of the original classical problem.

Nontrivial Question

Can we use this algorithm to solve classical matching?

• Yes, we can!

Corollary

Given any classical matching problem, we can find **all** stable matchings.

Nontrivial Question

Can we use this algorithm to solve classical matching?

• Yes, we can!

Key Idea Align student and college preferences!

Acknowledgments

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Image: A mathematical states of the state

Using Matching with Preferences over Colleagues

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Questions?

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April 11, 2009

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Extra Slides

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April 11, 2009

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Using Matching with Preferences over Colleagues

The Construction

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• One College: Z

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April 11, 2009 12 / 13

- One College: Z
- Two students: s_1, s_2

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- Two students: s_1, s_2
- Classical preference profiles \succ

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- One College: Z
- Two students: s_1, s_2
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• $Z \succ_{s_1} \emptyset$

- One College: Z
- Two students: s_1, s_2
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• $Z \succ_{s_i} \emptyset \ (i = 1, 2)$

•
$$\{s_1, s_2\} \triangleright_Z \{s_1\} \triangleright_Z \emptyset$$

• $(Z,) \triangleright_{s_1} (\emptyset,)$

- One College: Z
- Two students: s_1, s_2
- Classical preference profiles \succ

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- Nonclassical preference profiles ▷
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 - $(Z, \{s_1, s_2\}) \triangleright_{s_1} (Z, \{s_1\}) \triangleright_{s_1} (\emptyset, \emptyset)$
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Using Matching with Preferences over Colleagues

Complexity Analysis

Scott Duke Kominers (Harvard)

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Image: A mathematical states of the state

• $|\mathcal{P}| :=$ size of largest preference relation in \mathcal{P}

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 - \sim *input size* of our algorithm
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|P_{GS}| ~ baseline
|P_{EY}(P_{GS})| = O(|P_{GS}|²)
~ input size of our algorithm
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Question

Can we do better?

• $|\mathcal{P}| :=$ size of largest preference relation in \mathcal{P}

|P_{GS}| ~ baseline |P_{EY}(P_{GS})| = O(|P_{GS}|²) ~ input size of our algorithm ~ running time of the deferred acceptance algorithm

Open Question

Can we do better?