

Market Design Approaches to Inequality

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Society of Fellows, Harvard University, and
Becker Friedman Institute for Research in Economics, University of Chicago

Summer School on Socioeconomic Inequality
University of Chicago
July 17, 2013

Market Design Approaches to Inequality I: Balancing Fairness, Efficiency, and Incentives

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Overview

Now

- The Market Design Approach
- Design of School Choice Programs
- Cadet–Branch Matching; Eminent Domain

Later

- Design of Affirmative Action Mechanisms

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What is Market Design?

Application of economic principles and game theory to the design (or re-design) of market institutions.

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Theory \longrightarrow Practice \longrightarrow Evaluation

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e.g., no-trade theorems; nonexistence results
- 3 Working Within Existing Conditions (where possible/necessary)
e.g., existing policy goals
- 4 Organizing Market Function
e.g., strategy-proof mechanisms → accurate data

Some Key Concepts

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 - success requires *participation*
- ➌ Evaluation Criteria
 - vary from setting to setting; often depend on policy goals
- ➍ Flexibility
 - often crucial for market organizers

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The Setting

- Centralized assignment of K-12 public school seats.
- Students (i.e. their parents) are (potentially) strategic agents.
- School seats are “goods”; students have unit demand.
- Students’ priorities at schools are **exogenous**.

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- A **mechanism** φ assigns a match, given submitted preferences.

Basic Design Goals

- **Individual Rationality** (\sim participation)
 - No student wants to drop out (i.e. $\mu(i)P^i\emptyset$).

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 - If i envies j , then j has higher priority than i at $\mu(j)$ (i.e. $\mu(j)P^i\mu(i) \implies j\Pi^{\mu(j)}i$).

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 - Truthfulness is dominant (i.e. $\varphi(P^i, P^{-i})P^i\varphi(\bar{P}^i, P^{-i})$).
- **Pareto Efficiency**
- **Respect of (unambiguous) Improvements in Priority**

Backdrop: A Negative Result (Kesten, 2010)

Theorem

There is no Pareto efficient and strategy-proof mechanism that selects the Pareto efficient and stable match whenever such a match exists.

The Student-Optimal Stable Mechanism (SOSM)

Step 1

- Each student applies to his first-choice school.
- Each school tentatively “holds” its highest-priority applicants (up to capacity) and rejects all others.

Step $\ell \geq 2$

- Each student not currently “held” applies to his most-preferred school that has not yet rejected him.
- Each school “holds” its highest-priority applicants (up to capacity) and rejects all others.

★ Is stable and strategy-proof; is not Pareto efficient.

The Boston Mechanism

Step 1

- Each student applies to his first-choice school.
- Each school accepts its highest-priority applicants (up to capacity) and rejects all others.

Step $\ell \geq 2$

- Each not-yet-accepted student applies to his ℓ -th choice school.
- Each school accepts its highest-priority applicants (up to remaining capacity) and rejects all others.

-
- ★ Is Pareto efficient; is *neither* stable nor strategy-proof.
 - ★ Popular in practice – why?

Problems with The Boston Mechanism

Even if a student has very high priority at school c , he can lose his priority to students who have top-ranked school c !

For a better choice of your “first choice” school [...] consider choosing less popular schools.

(Introducing Boston Public Schools, 2004)

Problems with The Boston Mechanism

Even if a student has very high priority at school c , he can lose his priority to students who have top-ranked school c !

Make a realistic, informed selection on the school you list as your first choice. It's the cleanest shot you will get at a school, but if you aim too high you might miss.

*Here's why: If the random computer selection rejects your first choice, your chances of getting your second choice school are greatly diminished. That's because **you then fall in line behind everyone who wanted your second choice school as their first choice**. You can fall even farther back in line as you get bumped down to your third, fourth and fifth choices.*

(St. Petersburg Times, 2003)

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Even if a student has very high priority at school c , he can lose his priority to students who have top-ranked school c !

One school choice strategy is to find a school you like that is undersubscribed and put it as a top choice, OR, find a school that you like that is popular and put it as a first choice and find a school that is less popular for a “safe” second choice.

(West Zone Parents Group minutes, 2003)

Sincere vs. Sophisticated (Parents) (Pathak–Sönmez, 2008)

- Assume that the unsophisticated are truthful.
 - natural default behavior
 - suggested by anecdotes (Hastings–Kane–Staiger, 2005) and experimental evidence (Chen–Sönmez, 2006)
- Assume that the sophisticated best-respond.
- Consider the equilibrium ...

Sincere vs. Sophisticated (Parents) (Pathak-Sönmez, 2008)

- 1 In equilibrium under the Boston mechanism, sincere students lose their priorities to sophisticated students.
- 2 Sophisticated students never lose priority; sincere students may gain priority at the expenses of other sincere students.
- 3 (Coordinated) sophisticated students prefer Boston to SOSM.
- 4 Sophisticated students prefer that the sincere remain sincere.

Sincere vs. Sophisticated (Parents) (Pathak-Sönmez, 2008)

A strategy-proof algorithm “levels the playing field” by diminishing the harm done to parents who do not strategize or do not strategize well.

(BPS Strategic Planning Team, 2005)

School Admissions Reforms in the Last Decade

- New mechanisms, with direct consultation of economists:
 - 2003: New York City
 - 2005: Boston
- Mechanisms abandoned, without direct economist involvement:
 - 2007: England
 - 2009: Chicago
- Discussions about the vulnerability of mechanisms to manipulation played a key role in each of these reforms.
- *But not all reformers chose strategy-proof mechanisms.*

Chicago Sun-Times – November 12, 2009

8th-graders' shot at elite high schools better

Poring over data about eighth-graders who applied to the city's elite college preps, Chicago Public Schools officials discovered an alarming pattern. High-scoring kids were being rejected simply because of the order in which they listed their college prep preferences.

"I couldn't believe it," schools CEO Ron Huberman said. "It's terrible." CPS officials said Wednesday they have decided to let any eighth-grader who applied to a college prep for fall 2010 admission re-rank their preferences to better conform with a new selection system.

Previously, some eighth-graders were listing the most competitive college preps as their top choice, forgoing their chances of getting into other schools that would have accepted them if they had ranked those schools higher, an official said.

Under the new policy, Huberman said, a computer will assign applicants to the highest-ranked school they qualify for on their list.

"It's the fairest way to do it." Huberman told Sun-Times.

The Chicago School Choice Mechanisms

Old (“CHI⁴”)

Boston mechanism, *with forced preference list truncation* (down to four schools).

New (“SD⁴”)

SOSM, *with forced preference list truncation* (down to four schools).

- Urgent midstream change, yet *both are manipulable*.

Comparing Manipulable Mechanisms (Pathak–Sönmez, 2013)

Definition

- ① Mechanism φ is **as manipulable as** mechanism ψ if for any instance in which ψ is manipulable, φ is also manipulable.
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Theorem

CHI^4 (old) is more manipulable than SD^4 (new).

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Theorem

CH^4 (old) is as manipulable as any (weakly) stable mechanism.

Comparing Manipulable Mechanisms (Pathak–Sönmez, 2013)

- The last two results suggest that the new mechanism in Chicago is an improvement in terms of discouraging manipulation.
- However, requiring truncation is still sub-optimal—both in terms of efficiency and incentive compatibility.
- For the 2010–2011 school year, Chicago decided to increase the preference list length to 6, but the resulting mechanism is still manipulable (albeit less manipulable than SD^4).
- Similar design choices in New York and (throughout!) England.

The Boston Mechanism: Outlawed in England

Section 2.13: *In setting oversubscription criteria the admission authorities for all maintained schools must not: [...] give priority to children according to the order of other schools named as preferences by their parents, including 'first preference first' arrangements.*

(2007 School Admissions Code)

The Boston Mechanism: Outlawed in England



Additional Design Goals

- **Incentivize School Improvement** (Hatfield–Kojima–Narita, 2012)
- **True “Choice”** (Calsamiglia–Miralles, 2013)
- **“Cardinal” Efficiency** (Abdulkadiroğlu–Che–Yasuda, 2011)

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(But first. . .)

The Top Trading Cycles Mechanism

Step 1

- Assign each school c a “counter” κ_c which keeps track of the number of slots available at that school. Initially set $\kappa_c = q_c$.
- Each student “points to” his favorite school. Each school c points to the student who has the highest priority under Π^c .
- There is at least one cycle. Every student in a cycle is assigned a slot at the school he points to and is removed. The κ_c of each school c in a cycle is reduced by 1; if κ_c reaches 0, then c is also removed. Counters of other schools are unchanged.

Step $\ell \geq 2$

- Repeat Step 1 for the remaining “economy.”

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- ★ Is Pareto efficient and strategy-proof; is not stable.
 - ★ Somewhat unused in practice – why?

School Choice and School Competition

School choice advocates emphasize the effect of school choice on schools' incentives to improve themselves:

If we [...] implement choice among public schools, we unlock the values of competition in the educational marketplace. Schools that compete for students [...] will by virtue of their environment make those changes that allow them to succeed.

(Time for Results, National Governors' Association)

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[School choice will induce schools] to educate, to be responsive, to be efficient, and to innovate.

(Moe, 2008)

Improvement Incentives (Hatfield–Kojima–Narita, 2012)

Definition

A mechanism **respects improvements of school quality** if when students rank school c higher, c obtains a “better” set of students.

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A mechanism **respects improvements of school quality** if when students rank school c higher, c obtains a “better” set of students.

Bad News

- No stable mechanism (e.g., SOSM) respects improvements of school quality.
- No Pareto efficient mechanism (e.g., Boston, TTC) respects improvements of school quality.
- These negative results are quite general.

Improvement Incentives (Hatfield–Kojima–Narita, 2012)

Definition

A mechanism **approximately respects improvements of school quality** if for “almost all” preference profiles, no school is better off when students demote it in their rankings.

Good News

- Any stable mechanism (e.g., SOSM) approximately respects improvements of school quality.
- The Boston and TTC mechanisms do not approximately respect improvements of school quality.
- “Large market” results in tradition of Immorlica–Mahdian (2005) and Kojima–Pathak (2008).

Improvement Incentives (Hatfield–Kojima–Narita, 2012)

SOSM incentivizes improvement; Boston, TTC do not.

(More generally, market designers need to consider the impact of design on agents' long-term incentives!)

A Price-Theoretic Approach (Azevedo–Leshno, 2013)

In a continuum model, we can model the impact of increasing school quality directly:

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$$\frac{d\text{Composition}_c}{d\text{Quality}_c} = \underbrace{\text{Good Students who Move in on the Margin}}_{\text{Direct Effect}} + \underbrace{\text{Gains/Losses from Changing Other Schools' Selectivities}}_{\text{Market Power Effect}}$$

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- Making other schools less selective may harm school c !
(For generalization, see Veiga–Weyl (2012).)

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Even more, they emphasize the fact that school choice programs should actually enable *choice*.

School Choice is . . . a common sense idea that gives all parents the power and freedom to choose their child's education [...].

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Even more, they emphasize the fact that school choice programs should actually enable *choice*.

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Bad News

- In a large market with “neighborhood” priority and **agreement as to the worst school**, the probability that any student will be allocated to any good school that is not his or her neighborhood school is very low.

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Slightly Better News

- TTC does enable cross-neighborhood assignment, but does not help students who live in the bad neighborhood.

Pessimistic conclusion? No!

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- 3 Now, we can start to think about how design interacts with more classic economic questions (e.g., Calsamiglia–Martinez-Mora–Miralles (in prep.)).

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And as to getting students out of especially bad neighborhoods:

- Later today, we will incorporate affirmative action.

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ROTC Cadet–Branch Matching (Sönmez, 2013)

To increase officer retention, the Army recently introduced a “branch-of-choice” program, in which cadets may “bid” for priority.

- This system is a sort of “simplified auction” (technically fascinating, from matching-theoretic perspective).
- Today, we will focus on the inequality/diversity issues.

Diversity Concerns (Lim, 2009)

- Military leadership is demographically homogeneous: In 2006, only about 16% of officers were African American or Hispanic.
- Scarcity of minorities in combat arms branches is a barrier to improving diversity in the senior ranks.
- While 58% of white cadets' submitted first choices were in combat arms, only 31% of African American cadets' were.

Diversity Concerns (Lim, 2009)

On the one hand, minority cadets could truly prefer different career fields than white cadets. In this case, policy should focus on ways to make combat career fields more appealing to minorities. On the other hand, minorities may not really prefer support career fields but rather may reason that they lack the OML to get a more competitive career field [...] and may opt for their most-preferred Combat Support or Combat Service Support career field [...].

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Sound familiar?

Design Sketch (Sönmez, 2013)

Using the (full) cadet-optimal stable mechanism,¹ can solve these problems—and more!

- ★ Stable, strategy-proof, and improvement-respecting.

¹with well-chosen priority structure

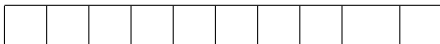
Why does priority structure matter? (Sönmez, 2013)

"20% in the complete OML [order of merit list] might actually be 28% in the 'Active Duty' OML, so make sure you make this mental conversion to the complete OML during your first three years. Or, just really screw up everything except for GPA, and get yourself into the 55% (from the top = 45%) where you get your choice of Branch... just kidding. But in all seriousness, why create a system of merit evaluation that takes a top 40% OML cadet and rewards him/her for purposely sabotaging things to go DOWN in the OML to below the 50% AD OML line[. . .]?"

(Service Academy Forums, 2012)

Holdout in the Assembly of Complements

- Ten people own (privately valued) homes



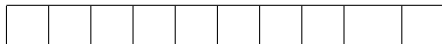
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 - Take-it-or-leave-it offers of $1, \dots, 10$ (total 55)?
 - $p(\text{sale}) = 10^{-10} = .0000000001$

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 - Take-it-or-leave-it offers of 8 (total 80)?
 - $p(\text{sale}) = \left(\frac{8}{10}\right)^{10} < .11$

Holdout in the Assembly of Complements

- Ten people own (privately valued) homes



- You want to buy their land and build a mall (worth 90)
 - All you know is that their values are uniformly distributed in $\{1, \dots, 10\}$ (expected total value 55)
- What should you do??
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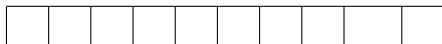
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2	3	4	5	6	7	8	9	10	1
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Basic Model

- Buyer has (private) value b for *aggregate plot*.
- Each seller i has (private) value v_i for *her land*.
- Each seller has expected share of total value s_i .
 - can be entirely exogenous or determined by buyer
 - s_i close to $v_i / (\sum_j v_j) \implies$ better property rights
- A *mechanism* is a transaction procedure.

Design Goals: Ideal

- ① Fully Efficient: mechanism captures all gains from trade
 - Sale $\iff b \geq \sum_i v_i \equiv V$
- ② Individually Rational: no seller sells for less than value
 - Sale \implies each seller i receives at least v_i
- ③ Budget-Balanced
 - No transfers to/from the market-maker

Design Goals: Proposal (K.-Weyl, 2012)

- ➊ Straightforward for Sellers: truthful play dominant (for sellers)
- ➋ Bilaterally Efficient: as efficient as bilateral trade
 - Sale $\iff o^*(b) \geq V$
- ➌ Partial Individual Rationality
 - Approximate IR: seller i receives at least $\frac{s_i(V - v_i)}{1 - s_i}$
 - Collective IR: community not forced to sell for less than V
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Concordance among Holdouts (K.-Weyl, 2012)

- ① Introduce holdout as a market design problem
 - Goals – straightforwardness, bilateral efficiency, approximate IR
- ② Propose solution approach
 - “Concordance” – divide profits according to *ex ante* shares s_i
- ③ Investigate when competition offsets complementarity
 - Combinatorial holdout – clusters and repacking

Concordance among Holdouts (K.-Weyl, 2012)

- In Concordance Mechanisms:
 - 1 Sellers i divide offer o into previously-specified shares $s_i o$.
 - 2 Each seller pays a Pigouvian tax for externalities.

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Concordance among Holdouts (K.-Weyl, 2012)

- Problem: **choice of collective decision-making procedure**

Concordance among Holdouts (K.-Weyl, 2012)

- Problem: **choice of collective decision-making procedure**
 - VCG – vulnerable to collusion, not budget-balanced
 - expected externality, voting – require distributional information
 - legal recourse – buyers can exploit coercive power
 - quadratic vote buying (Weyl, in preparation) – . . . ?

Overview

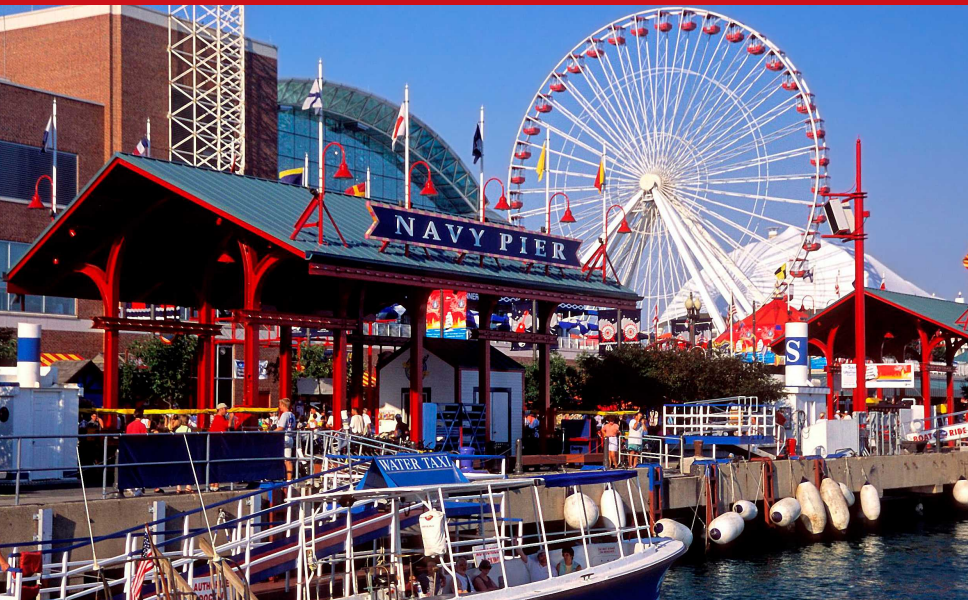
Now

- The Market Design Approach
- Design of School Choice Programs
- Cadet–Branch Matching; Eminent Domain

Later

- Design of Affirmative Action Mechanisms

\pause



Market Design Approaches to Inequality II: Design of Affirmative Action Mechanisms

Scott Duke Kominers

Society of Fellows, Harvard University, and
Becker Friedman Institute for Research in Economics, University of Chicago

Summer School on Socioeconomic Inequality
University of Chicago
July 17, 2013

Overview

Earlier

- Balancing Fairness, Efficiency, and Incentives

Now

- Quota-Based Mechanisms
- Minority Reserves
- Slot-Specific Priorities

The Setting

- Centralized assignment of K-12 public school seats.
- Students (i.e. their parents) are (potentially) strategic agents.
- School seats are “goods”; students have unit demand.
- Students’ priorities at schools are **exogenous**.

Basic Theory (Abdulkadiroğlu–Sönmez, 2003)

- $I \sim$ set of students
- $C \sim$ set of schools
- $P^i \sim$ preference ranking of $i \in I$ over schools (and \emptyset)
- $\Pi^c \sim$ priority ranking of $c \in C$ over students
- $q_c \sim$ total capacity of $c \in C$
- A **match** μ specifies an assignment of students to schools.
(must respect capacities – $|\mu(c)| \leq q_c$)
- A **mechanism** φ assigns a match, given submitted preferences.

Basic Design Goals

- **Individual Rationality** (\sim participation)
 - No student wants to drop out (i.e. $\mu(i)P^i\emptyset$).
- **Elimination of Justified Envy** (\sim stability)
 - If i envies j , then j has higher priority than i at $\mu(j)$ (i.e. $\mu(j)P^i\mu(i) \implies j\Pi^{\mu(j)}i$).
- **Strategy-proofness**
 - Truthfulness is dominant (i.e. $\varphi(P^i, P^{-i})P^i\varphi(\bar{P}^i, P^{-i})$).
- **Pareto Efficiency**
- **Respect of (unambiguous) Improvements in Priority**

Now

Use insights from matching theory to inform the design of affirmative action mechanisms.

Overview

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- Balancing Fairness, Efficiency, and Incentives

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Backdrop

*Affirmative Action refers to positive steps aimed at increasing the inclusion of historically excluded groups in employment, education and business. Such steps are not designed to offer preferential treatment to, or exclude from participation, any group. To the contrary, **Affirmative Action policies are intended to promote access for the traditionally underrepresented through heightened outreach and efforts at inclusion.***

(American Association for Affirmative Action)

Backdrop

“Affirmative action” means positive steps taken to increase the representation of women and minorities in areas of employment, education, and business from which they have been historically excluded. When those steps involve [...] selection on the basis of race, gender, or ethnicity [...] affirmative action generates intense controversy.

(Stanford Encyclopedia of Philosophy)

Backdrop

Forty years ago, as the United States experienced the civil rights movement [...]. After a full generation [...] a plethora of government-enforced diversity policies have marginalized many white workers.

(Sen. James Webb, 2010)

The Civil Rights Act of 1964 gave the federal government unprecedented power over the hiring, employee relations, and customer service practices of every business in the country. The result was a massive violation of the rights of private property and contract, which are the bedrocks of free society.

(Rep. Ron Paul, 2004)

Background

While there is (heated) disagreement about the value of affirmative action; there is little disagreement about what affirmative action actually *does*.

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While there is (heated) disagreement about the value of affirmative action; there is little disagreement about what affirmative action actually *does*.

But...

Popular “majority quota”-based affirmative action policies can hurt *every minority student*.

Extended Model (Kojima, 2012)

- $I \sim$ set of students (each $i \in I$ has type M or m)
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Definition

A match is **stable** (in the presence of quotas) if

- ① it is **individually rational** – $\mu(i)P^i\emptyset$ for all $i \in I$ – and
- ② it is **unblocked** – if $cP^i\mu(i)$, then either
 - $|\mu(c)| = q_c$ and $j \Pi^c i$ for all $j \in \mu(c)$, or
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(This looks really complicated. . .)

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Definition

A mechanism is **stable** if it always selects stable outcomes.

Affirmative Action with Majority Quotas (Kojima, 2012)

Definition

A mechanism **respects the spirit of quota-based affirmative action** if when majority quotas are decreased ($q_c^M \rightarrow \tilde{q}_c^M < q_c^M$), minority outcomes improve.

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Bad News

- No stable mechanism respects the spirit of quota-based affirmative action. (The quota outcome can be Pareto inferior!)
- Quotas \sim unpredictable. (They *can* cause Pareto improvement.)
- Similar results for {“priority-based” affirmative action, TTC}.

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(But wait. . .)

Re-examining the Example

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Key Observation (I)

*The reason that a quota for majority students can have adverse effects on minority students is simple. Consider a situation in which a school c is mostly desired by majorities. Then **having a majority quota for c decreases the number of majority students that can be assigned to c even if there are empty seats. This, in turn, increases the competition for other schools and thus can even make the minority students worse off.***

(Hafalir–Yenmez–Yildirim, 2013)

Overview

Earlier

- Balancing Fairness, Efficiency, and Incentives

Now

- Quota-Based Mechanisms
- Minority Reserves
- Slot-Specific Priorities

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Key Observation (II)

The number of minority students preferring a school to another is not known a priori by the policymakers. Even most intelligent guesses of quota levels will be prone to small deviations in minority students' realized desire to attend a particular school, which might cascade inefficiencies throughout the system. [...] Moreover, these quotas are usually set by third parties such as courts or school districts, which means that they cannot be readjusted easily if schools have empty seats.

(Hafalir–Yenmez–Yildirim, 2013)

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 - $t(i) = m$ and all $j \in \mu(c)$ have higher priority than i ,
 - $t(i) = M$, c 's reserved slots are not full, and all $j \in \mu(c)$ have higher priority at c than i , or
 - $t(i) = M$, c 's reserved slots are full, and all $j \in (\mu(c) \cap I_M)$ have higher priority at c than i .

Minority Reserves “Work” (Hafalir–Yenmez–Yildirim, 2013)

- 1 For any match μ stable under quotas q^M , there exists a match stable under reserves $r^m = q - q^M$ that Pareto improves on μ .
- 2 Minority students never (Pareto) prefer the SOSM without affirmative action to the SOSM with minority reserves.
- 3 Under natural conditions, minority students (Pareto) prefer SOSM with reserves to the SOSM without affirmative action.
- 4 Similar results hold for “TTC with minority reserves.”

Simulations Say More (Hafalir–Yenmez–Yildirim, 2013)

- 1 Reserves improve minority welfare (but can hurt majorities).
- 2 SOSM with minority reserves “significantly” Pareto dominates SOSM with majority quotas (for all students).
- 3 Quota-based mechanism outcomes are sensitive to quota size.
- 4 Students on average prefer TTC to SOSM for all affirmative action policies.

Related Approaches

- **Regional Quotas** (Kamada–Kojima, 2011)
- **“Complex Constraints”** (Westkamp, 2012)
- **Slot-Specific Priorities** (K.–Sönmez, 2012)

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\pause

Last component of the talk—current/ongoing research.

\pause

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(Please mind the notation.)

So far...

Theory \longrightarrow Practice \longrightarrow Evaluation

Meanwhile...

Theory \longrightarrow Practice \longrightarrow Evaluation

Now...

Theory \longrightarrow Practice \longrightarrow Evaluation

Now...

Theory ← Evaluation

The Big Idea

Suppose that a school has slots reserved for minorities and slots reserved for women. . . .

Socioeconomic Affirmative Action in Chicago

The Setting:

- Students \leftrightarrow Elite Public High Schools

The Problem:

- 15% of slots are reserved for each class ($t \in \{4, 3, 2, 1\}$).
- \Rightarrow Priorities vary across slots.

Chicago's Solution:

- Divide each school into (five) sub-schools;
- Run the student-optimal stable mechanism, filling “open” sub-schools before “reserved” ones.

Thought Experiment

Chicago School Choice:

Test scores \Rightarrow global priority π ; some slots have minority reserves.

$$S^0 : \{4, 3, 2, 1\}$$

$$S^4 : 4 \succ \{3, 2, 1\}$$

$$S^3 : 3 \succ \{4, 2, 1\}$$

$$S^2 : 2 \succ \{4, 3, 1\}$$

$$S^1 : 1 \succ \{4, 3, 2\}$$

Facts

- Minorities (e.g., Tier 1) have systematically low test scores.
- 16,372 students compete for 4,270 elite high school slots.

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**Total
changeover:
766 slots
 $\approx 18\%$**

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Impact of Chicago Slot-Precedence Order (I)

Simulation confirms our intuition:

Using preferences submitted to the (current) Chicago school choice mechanism, we can simulate alternate mechanisms.

- Recall: 16,372 students; 4,270 elite high school slots.

Current Mechanism:

$$S^{\circ} : \{4, 3, 2, 1\}$$

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Counterfactual:

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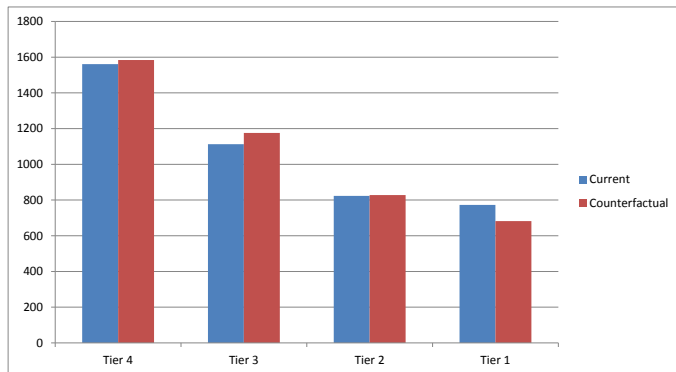
**Total
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Impact of Chicago Slot-Precedence Order (II)

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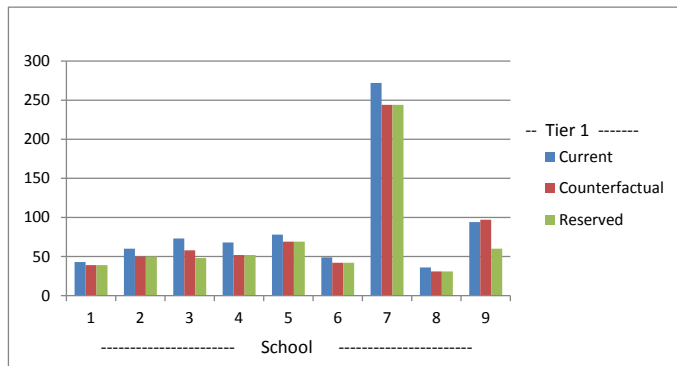


Impact of Chicago Slot-Precedence Order (III)

Simulation confirms our intuition:

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Model Overview

- Affirmative Action in Chicago
- Neighborhood Priority in Boston
- Cadet–Branch Matching

Problem:

Priorities vary across slots.

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Problem:

Priorities vary across slots.

We Give

A general and unified framework for these applications in which:

- Key substitutability conditions do not hold.
- Agent-optimal stable outcomes may not exist.
- The cumulative offer mechanism is nevertheless *stable*, *strategy-proof*, and *improvement-respecting*.

Model (Informal)

- A many-to-one matching model (with contracts).
- *Agents* have preferences over *contracts* with *branches*.
- Each branch has *slots* that can be assigned contracts.
- Each slot
 - can hold at most one contract, and
 - has its own priority order.
- Slots are filled sequentially, according to *precedence order*.

Model (Formal)

- Set I of agents; set B of branches.
- Set $X \subseteq I \times B \times T$ of contracts.

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- Set I of agents; set B of branches.
- Set $X \subseteq I \times B \times T$ of contracts.
- $P^i \sim$ preferences of i over $X_i \equiv \{x \in X : i(x) = i\}$.
 - Choice C^i defined by maximization.

Model (Formal)

- Set I of agents; set B of branches.
- Set $X \subseteq I \times B \times T$ of contracts.
- $P^i \sim$ preferences of i over $X_i \equiv \{x \in X : i(x) = i\}$.
 - Choice C^i defined by maximization.
- Set S^b of slots at branch b .
- Order of slot-precedence \triangleright^b over S^b .

Model (Formal)

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 - Choice C^i defined by maximization.
- Set S^b of slots at branch b .
- **Order of slot-precedence** \triangleright^b over S^b .
- $\Pi^s \sim$ priorities of $s \in S^b$ over $X_b \equiv \{x \in X : b(x) = b\}$.
 - Choice C^b defined by **\triangleright^b -sequential maximization**.

\triangleright^b -sequential Maximization: An Example

- $I = \{i, j, k\}$; $B = \{b\}$; $S^b = \{s_b^1 \triangleright^b s_b^2\}$; $X = \{i_1, j_1, k_1\}$.

$$\Pi^{s_b^1} : i_1 \succ j_1 \succ k_1 \succ \emptyset$$

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Solution Concept

Definition

An outcome $Y \subseteq X$ is **stable** if it is

① **Individually Rational:**

- $C^i(Y) = Y_i$ for all $i \in I$;
- $C^b(Y) = Y_b$ for all $b \in B$.

② **Unblocked:** There does not exist a nonempty **blocking set** $Z \not\subseteq Y$ such that

- $Z_i \subseteq C^i(Y \cup Z)$ for all $i \in i(Z)$;
- $Z_b \subseteq C^b(Y \cup Z)$ for all $b \in b(Z)$.

The Cumulative Offer Process (I)

Step 1

- ➊ One agent “proposes” her first-choice contract, x .
- ➋ $A_2^{b(x)} = \{x\}$; $A_2^b \equiv \emptyset$ for $b \neq b(x)$.
- ➌ Each branch b “holds” $C^b(A_2^b)$.
 - I.e. if x is acceptable to some $s \in S^{b(x)}$, then $b(x)$ holds x .

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Step $\ell \geq 2$

- ➊ Some agent for whom no contract is held proposes a contract y that has not yet been proposed.
- ➋ $A_{\ell+1}^{b(y)} = A_\ell^{b(y)} \cup \{y\}$; $A_{\ell+1}^b \equiv A_\ell^b$ for $b \neq b(y)$.
- ➌ Each branch b holds $C^b(A_{\ell+1}^b)$.

The Cumulative Offer Process (II)

Definition

The **cumulative offer mechanism** Φ_{Π} imposes the outcome of the cumulative offer process under priorities Π and submitted preferences.

Central Result

Theorem

The cumulative offer mechanism Φ_{Π} is stable and strategy-proof.

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Theorem

The cumulative offer mechanism Φ_{Π} is stable and strategy-proof.

(Observation: This *looks* like a standard result.)

Opposition of *Agents'* Interests

Example

- $I = \{i, j, k\}$; $B = \{b\}$; $S^b = \{s_b^1, s_b^2\}$; $X = \{i_1, i_2, j_1, j_2, k_1, k_2\}$.

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- Stable outcomes when $s_b^1 \triangleright^b s_b^2$: $Y \equiv \{j_2, i_1\}$ and $Y' \equiv \{i_2, j_1\}$.
- i prefers Y ; j prefers Y' .

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Summary of Results

Theorem

The cumulative offer mechanism

- ① *is stable and strategy-proof,*
- ② *is independent of proposal order, and*
- ③ *respects unambiguous improvements in agent priority.*

Summary of Results

Theorem

The cumulative offer mechanism

- 1 *is stable and strategy-proof,*
- 2 *is independent of proposal order, and*
- 3 *respects unambiguous improvements in agent priority.*

Applications

- Minority reserves (Hafalir–Yenmez–Yildirim (2013)).
- Impact of precedence in the presence of uncertainty.
- New approaches to {Chicago,Boston} school choice.
- Impact of slot-precedence in cadet–branch matching.

School Choice with Minority Reserves

Proposition (Hafalir–Yenmez–Yildirim (2013))

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- *selects the student-optimal stable outcome, and*
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School Choice with Minority Reserves

Proposition (Hafalir–Yenmez–Yildirim (2013))

- ① *In the presence of minority reserves, the cumulative offer mechanism*
 - *selects the student-optimal stable outcome, and*
 - *is strategy-proof.*
- ② *Given a vector q^M of majority quotas, set $r_b^m = L^b - q_b^M$ for each $b \in B$, and let Y be an outcome which is stable under the priorities Π induced by the quotas q^M and tiebreakers π . Either:*
 - ① *Y is stable under the priorities $\bar{\Pi}$ induced by π and reserves r^m , or*
 - ② *there exists an outcome Z which is stable under priorities $\bar{\Pi}$ and Pareto dominates Y .*

Response to Uncertainty

$p = \frac{1}{2}$: minorities score well

$p = \frac{1}{2}$: minorities score badly

$s^{m1} : \boxed{m_1} \succ m_2 \succ m_3 \succ \dots$

$s^{m2} : m_1 \succ \boxed{m_2} \succ m_3 \succ \dots$

$s^{o1} : \boxed{M_1} \succ M_2 \succ m_1 \succ m_2 \succ M_3 \succ M_4 \succ \dots$

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$s^{o3} : M_1 \succ M_2 \succ m_1 \succ m_2 \succ \boxed{M_3} \succ M_4 \succ \dots$

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$$E[\text{number of minorities admitted}] = 2$$

$$\text{Var}[\text{number of minorities admitted}] = 0$$

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$$E[\text{number of minorities admitted}] = 2$$

$$\text{Var}[\text{number of minorities admitted}] = 1$$

An Engineering Problem

What would an “intermediate” mechanism for the Chicago school choice program look like?

An Engineering Problem

Current

Open

Reserved

An Engineering Problem

Current

Open

Reserved

“Intermediate”

Open

Reserved

Open

Reserved

⋮

Open

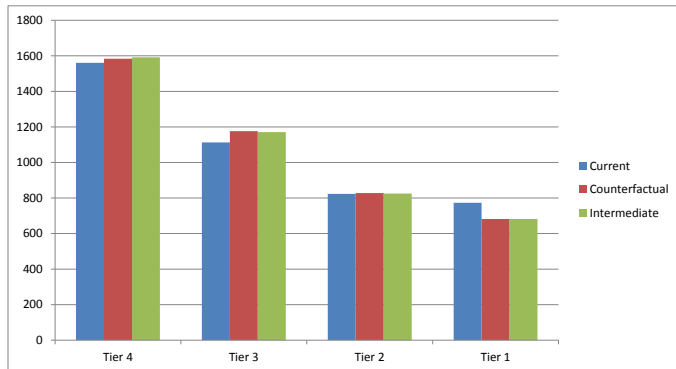
Reserved

Chicago with an “Intermediate” Mechanism

Simulation yields an (initially) surprising result:

Using preferences submitted to the (current) Chicago school choice mechanism, we can simulate alternate mechanisms.

- Recall: 16,372 students; 4,270 elite high school slots.



Socioeconomic Affirmative Action in Chicago

The Setting:

- Students \leftrightarrow Elite Public High Schools

The Problem:

- 15% of slots are reserved for each class ($t \in \{4, 3, 2, 1\}$).
- \Rightarrow Priorities vary across slots.

Chicago's Solution:

- Divide each school into (five) sub-schools;
- Run the student-optimal stable mechanism, filling “open” sub-schools before “reserved” ones.

Neighborhood Priority in Boston (Dur-Pathak-K.-Sönmez, 2013)

The Setting:

- Students \leftrightarrow K-12 Public Schools

The Problem:

- 50% of slots give “walk-zone priority.”
- \Rightarrow Priorities vary across slots.

Boston's Solution:

- Divide each school into (two) sub-schools;
- Run the student-optimal stable mechanism, filling “walk-zone” sub-schools before “open” ones.

Neighborhood Priority in Boston: An Example

■ π_1

■ π_2

□ π_3

□ π_4

■ \sim walk-zone slot

□ \sim open slot

Neighborhood Priority in Boston: An Example

■ w_1

■ i_1

□ w_2

□ i_2

lottery expectation

■ \sim walk-zone slot

□ \sim open slot

Neighborhood Priority in Boston: An Example

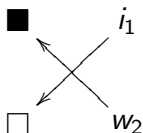


current mechanism
(first stage)

■ \sim walk-zone slot

□ \sim open slot

Neighborhood Priority in Boston: An Example



current mechanism
(second stage)

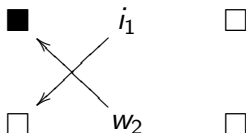
■ \sim walk-zone slot

□ \sim open slot

Neighborhood Priority in Boston: An Example

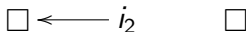


alternate mechanism



■ \sim walk-zone slot

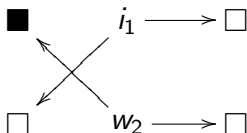
□ \sim open slot



Neighborhood Priority in Boston: An Example

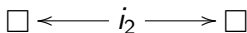


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■ \sim walk-zone slot

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Neighborhood Priority in Boston: Backdrop

“Something stands in the way of taking our [public school] system to the next level: a student assignment process that ships our kids to schools across our city. Pick any street. A dozen children probably attend a dozen different schools. Parents might not know each other; children might not play together. They can’t carpool, or study for the same tests. [...] Boston [needs] a radically different school assignment process—one that puts priority on children attending schools closer to their homes.”

(Boston Mayor Thomas Menino, 2012)

Neighborhood Priority in Boston: Theory

Proposition

Replacing an open slot at school b with a walk-zone slot (fixing precedence) weakly increases the number of walk-zone students assigned to b under the SOSM.

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Proposition

Lowering the precedence of a walk-zone slot of school b weakly increases the number of walk-zone students assigned to b under the SOSM.

Neighborhood Priority in Boston: Debate

"MIT tells us that so many children in the walk zones of high demand schools flood the pool of applicants, and that children in these walk zones get in in higher numbers, so walk zone priority doesn't really matter.

Maybe, that is true. But if removing the walk zone priority doesn't change anything, why change it all?"

(City Councillor in charge of education, 2013)

Neighborhood Priority in Boston: Outcome

"After viewing the final MIT and BC presentations on the way the walk zone priority actually works, it seems to me that it would be unwise to add a second priority to the Home-Based model by allowing the walk zone priority be carried over."

(Boston School Superintendent Carol R. Johnson, 2013)

Neighborhood Priority in Boston: Outcome

"Leaving the walk zone priority to continue as it currently operates is not a good option. We know from research that it does not make a significant difference the way it is applied today: although people may have thought that it did, the walk zone priority does not in fact actually help students attend schools closer to home. The External Advisory Committee suggested taking this important issue up in two years, but I believe we are ready to take this step now. We must ensure the Home-Based system works in an honest and transparent way from the very beginning."

(Boston School Superintendent Carol R. Johnson, 2013)

Neighborhood Priority in Boston: Outcome

Boston School Committee approves new student-assignment system

03/14/2013 2:23 AM

[E-mail](#) | [Print](#) | [Comments \(23\)](#)



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336



By James Vaznis, Globe Staff

The Boston School Committee, in a momentous vote Wednesday, scrapped a school assignment plan developed under court-ordered desegregation almost a quarter century ago and embraced a new system that seeks to allow more students to attend schools closer to home.

Starting in fall 2014, the city will do away with three sprawling assignment zones that the School Department has operated since 1989, each of which offers about two dozen school choices.

Under the new policy, a computer algorithm will generate a list of at least six schools from which parents will be able to choose based on a variety of factors, such as distance from school, school capacity, and MCAS performance. At least four of the school choices will be of medium or high quality.

The committee also eliminated the so-called walk-zone preference for students — within about a mile of a school. Such a policy can benefit students who live near a high-performing school to the detriment of others who do not have such a school nearby.

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Overview

Earlier

- Balancing Fairness, Efficiency, and Incentives

Now

- Quota-Based Mechanisms
- Minority Reserves
- Slot-Specific Priorities

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- Balancing Fairness, Efficiency, and Incentives

Now

- Quota-Based Mechanisms
- Minority Reserves
- Slot-Specific Priorities

What is Market Design?

Application of economic principles and game theory to the design (or re-design) of market institutions.

What is Market Design?

- 1 Economic Engineering
e.g., improving incentives; “leveling the playing field”
- 2 Working Around Impossibility Results
e.g., no-trade theorems; nonexistence results
- 3 Working Within Existing Conditions (where possible/necessary)
e.g., existing policy goals
- 4 Organizing Market Function
e.g., strategy-proof mechanisms → accurate data

Some Key Concepts

- ➊ Strategy-proofness (vs. Manipulability)
 - essential for ensuring simplicity; not always achievable
- ➋ Market Thickness
 - success requires *participation*
- ➌ Evaluation Criteria
 - vary from setting to setting; often depend on policy goals
- ➍ Flexibility
 - often crucial for market organizers

Why do we care?

Balancing fairness, efficiency, and incentives can be *hard*.

How is market design related to inequality?

Effective design can reduce frictions and help ensure equal access to the benefits of the market.

Where has market design been effective?

Many of our successes thus far have been in self-contained markets, with institution-driven market failures.

Where do we go from here?

New applications (e.g., resource alloc., climate change)!

Where do we go from here?

Partnerships/methods for evaluating existing designs. . . ?

Where do we go from here?

Econometrics attentive to the mechanisms in use...?

Where do we go from here?

Links with {labor, public, . . .} economics. . . ?

Who should practice market design?

Maybe you!

Who should practice market design?

