Market Design Approaches to Inequality I: Balancing Fairness, Efficiency, and Incentives

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Overview

Today

- The Market Design Approach
- Design of School Choice Programs
- Cadet–Branch Matching; Eminent Domain

Tomorrow

- Design of Affirmative Action Mechanisms
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What is Market Design?

Application of economic principles and game theory to the design (or re-design) of market institutions.
What is Market Design?

Theory $\rightarrow$ Practice $\rightarrow$ Evaluation
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Theory → Practice → Evaluation
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e.g., improving incentives; “leveling the playing field”
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2. Working Around Impossibility Results
   e.g., no-trade theorems; nonexistence results
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   - e.g., improving incentives; “leveling the playing field”

2. **Working Around Impossibility Results**
   - e.g., no-trade theorems; nonexistence results

3. **Working Within Existing Conditions (where possible/necessary)**
   - e.g., existing policy goals
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   e.g., improving incentives; “leveling the playing field”

2. Working Around Impossibility Results
   e.g., no-trade theorems; nonexistence results

3. Working Within Existing Conditions (where possible/necessary)
   e.g., existing policy goals

4. Organizing Market Function
   e.g., strategy-proof mechanisms → accurate data
Some Key Concepts

1. **Strategy-proofness (vs. Manipulability)**
   - essential for ensuring simplicity; not always achievable
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   - vary from setting to setting; often depend on policy goals
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   - essential for ensuring simplicity; not always achievable

2. Market Thickness
   - success requires *participation*

3. Evaluation Criteria
   - vary from setting to setting; often depend on policy goals

4. Flexibility
   - often crucial for market organizers
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The Setting

- Centralized assignment of K-12 public school seats.
- Students (i.e. their parents) are (potentially) strategic agents.
- School seats are “goods”; students have unit demand.
- Students’ priorities at schools are *exogenous*. 
Basic Theory (Abdulkadiroğlu–Sönmez, 2003)

- \( I \sim \) set of students
- \( C \sim \) set of schools
Basic Theory \cite{Abdulkadiro˘glu–Sönmez, 2003}

- \( I \sim \) set of students
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- \( P^i \sim \) preference ranking of \( i \in I \) over schools (and \( \emptyset \))
Basic Theory  (Abdulkadiroğlu–Sönmez, 2003)

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- $q_c \sim$ total capacity of $c \in C$
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A **match** $\mu$ specifies an assignment of students to schools. 
(must respect capacities $- |\mu(c)| \leq q_c$)
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- A **match** $\mu$ specifies an assignment of students to schools. 
  (must respect capacities – $|\mu(c)| \leq q_c$)

- A **mechanism** $\varphi$ assigns a match, given submitted preferences.
Basic Design Goals

- **Individual Rationality** ($\sim$ participation)
  - No student wants to drop out (i.e. $\mu(i)P^i\emptyset$).

- Elimination of Justified Envy ($\sim$ stability)
  - If $i$ envies $j$, then $j$ has higher priority than $i$ at $\mu(j)$ (i.e. $\mu(j)P\mu(i) \Rightarrow j\not\in\mu(i)$).

- Strategy-proofness
  - Truthfulness is dominant (i.e. $\phi(P^i, P^{-i})P^i \phi(\overline{P}^i, P^{-i})$).

- Pareto Efficiency
  - Respect of (unambiguous) Improvements in Priority
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  - Truthfulness is dominant (i.e. $\varphi(P^i, P^{-i})P^i\varphi(\bar{P}^i, P^{-i})$).

- **Pareto Efficiency**

- **Respect of (unambiguous) Improvements in Priority**
Theorem

There is no Pareto efficient and strategy-proof mechanism that selects the Pareto efficient and stable match whenever such a match exists.
The Student-Optimal Stable Mechanism (SOSM)

Step 1
- Each student applies to his first-choice school.
- Each school tentatively “holds” its highest-priority applicants (up to remaining capacity) and rejects all others.

Step \( \ell \geq 2 \)
- Each student not currently “held” applies to his most-preferred school that has not yet rejected him.
- Each school “holds” its highest-priority applicants (up to remaining capacity) and rejects all others.

- Is stable and strategy-proof; is not Pareto efficient.
The Boston Mechanism

Step 1

- Each student applies to his first-choice school.
- Each school accepts its highest-priority applicants (up to capacity) and rejects all others.

Step $\ell \geq 2$

- Each not-yet-accepted student applies to his $\ell$-th choice school.
- Each school accepts its highest-priority applicants (up to remaining capacity) and rejects all others.

- Is Pareto efficient; is neither stable nor strategy-proof.
- Popular in practice – why?
Problems with The Boston Mechanism

Even if a student has very high priority at school $c$, he can lose his priority to students who have top-ranked school $c$!

*For a better choice of your “first choice” school [...] consider choosing less popular schools.*

*(Introducing Boston Public Schools, 2004)*
Problems with The Boston Mechanism

Even if a student has very high priority at school $c$, he can lose his priority to students who have top-ranked school $c$!

Make a realistic, informed selection on the school you list as your first choice. It’s the cleanest shot you will get at a school, but if you aim too high you might miss.

Here’s why: If the random computer selection rejects your first choice, your chances of getting your second choice school are greatly diminished. That’s because you then fall in line behind everyone who wanted your second choice school as their first choice. You can fall even farther back in line as you get bumped down to your third, fourth and fifth choices.

(St. Petersburg Times, 2003)
Problems with The Boston Mechanism

Even if a student has very high priority at school \( c \), he can lose his priority to students who have top-ranked school \( c \)!

One school choice strategy is to find a school you like that is undersubscribed and put it as a top choice, OR, find a school that you like that is popular and put it as a first choice and find a school that is less popular for a “safe” second choice.

(West Zone Parents Group minutes, 2003)
Sincere vs. Sophisticated (Parents) (Pathak–Sönmez, 2008)

- Assume that the unsophisticated are truthful.
  - natural default behavior
  - suggested by anecdotes (Hastings–Kane–Staiger, 2005) and experimental evidence (Chen–Sönmez, 2006)

- Assume that the sophisticated best-respond.

- Consider the equilibrium . . .
Sincere vs. Sophisticated (Parents)  
(Pathak–Sönmez, 2008)

1. In equilibrium under the Boston mechanism, sincere students lose their priorities to sophisticated students.

2. Sophisticated students never lose priority; sincere students may gain priority at the expenses of other sincere students.

3. (Coordinated) sophisticated students prefer Boston to SOSM.

4. Sophisticated students prefer that the sincere remain sincere.
Sincere vs. Sophisticated (Parents) (Pathak–Sönmez, 2008)

A strategy-proof algorithm “levels the playing field” by diminishing the harm done to parents who do not strategize or do not strategize well.

(BPS Strategic Planning Team, 2005)
School Admissions Reforms in the Last Decade

- New mechanisms, with direct consultation of economists:
  - 2003: New York City
  - 2005: Boston

- Mechanisms abandoned, without direct economist involvement:
  - 2007: England
  - 2009: Chicago

- Discussions about the vulnerability of mechanisms to manipulation played a key role in each of these reforms.

- *But not all reformers chose strategy-proof mechanisms.*
Poring over data about eighth-graders who applied to the city’s elite college preps, Chicago Public Schools officials discovered an alarming pattern. High-scoring kids were being rejected simply because of the order in which they listed their college prep preferences.

“I couldn’t believe it,” schools CEO Ron Huberman said. “It’s terrible.” CPS officials said Wednesday they have decided to let any eighth-grader who applied to a college prep for fall 2010 admission re-rank their preferences to better conform with a new selection system.

Previously, some eighth-graders were listing the most competitive college preps as their top choice, forgoing their chances of getting into other schools that would have accepted them if they had ranked those schools higher, an official said.

Under the new policy, Huberman said, a computer will assign applicants to the highest-ranked school they quality for on their list.

“It’s the fairest way to do it.” Huberman told Sun-Times.
The Chicago School Choice Mechanisms

Old ("CHI$^4$")
Boston mechanism, *with forced preference list truncation* (down to four schools).

New ("Sd$^4$")
SOSM, *with forced preference list truncation* (down to four schools).

- Urgent midstream change, yet *both are manipulable*. 
Comparing Manipulable Mechanisms (Pathak–Sönmez, forth.)

Definition

1. Mechanism $\varphi$ is as manipulable as mechanism $\psi$ if for any instance in which $\psi$ is manipulable, $\varphi$ is also manipulable.

2. Mechanism $\varphi$ is more manipulable than mechanism $\psi$ if
   - $\varphi$ is at least as manipulable as $\psi$, and
   - there is an instance in which $\varphi$ is manipulable and $\psi$ is not.

Theorem

Chi$_4$ (old) is more manipulable than Sd$_4$ (new).
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Theorem

$\text{CHI}^4$ (old) is more manipulable than $\text{SD}^4$ (new).
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Theorem

$\text{Chi}^4$ (old) is as manipulable as any (weakly) stable mechanism.
The last two results suggest that the new mechanism in Chicago is an improvement in terms of discouraging manipulation.

However, requiring truncation is still sub-optimal—both in terms of efficiency and incentive compatibility.

For the 2010–2011 school year, Chicago decided to increase the preference list length to 6, but the resulting mechanism is still manipulable (albeit less manipulable than $SD^4$).

Similar design choices in New York and (throughout!) England.
Section 2.13: *In setting oversubscription criteria the admission authorities for all maintained schools must not: [...] give priority to children according to the order of other schools named as preferences by their parents, including ‘first preference first’ arrangements.*

*(2007 School Admissions Code)*
The Boston Mechanism: Outlawed in England
Additional Design Goals

- **Incentivize School Improvement** (Hatfield–Kojima–Narita, 2012)

- **True “Choice”** (Calsamiglia–Miralles, 2012)

- **“Cardinal” Efficiency** (Abdulkadiroğlu–Che–Yasuda, 2011)
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(But first...)
The Top Trading Cycles Mechanism

Step 1

- Assign each school $c$ a “counter” $\kappa_c$ which keeps track of the number of slots available at that school. Initially set $\kappa_c = q_c$.
- Each student “points to” his favorite school. Each school $c$ points to the student who has the highest priority under $\Pi^c$.
- There is at least one cycle. Every student in a cycle is assigned a slot at the school he points to and is removed. The $\kappa_c$ of each school $c$ in a cycle is reduced by 1; if $\kappa_c$ reaches 0, then $c$ is also removed. Counters of other schools are unchanged.

Step $\ell \geq 2$

- Repeat Step 1 for the remaining “economy.”
The Top Trading Cycles Mechanism

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- Is Pareto efficient and strategy-proof; is not stable.

- Somewhat unused in practice – why?
School Choice and School Competition

School choice advocates emphasize the effect of school choice on schools’ incentives to improve themselves:

*If we [...] implement choice among public schools, we unlock the values of competition in the educational marketplace. Schools that compete for students [...] will by virtue of their environment make those changes that allow them to succeed.*

*(Time for Results, National Governors’ Association)*
School choice advocates emphasize the effect of school choice on schools’ incentives to improve themselves:

\[\text{School choice will induce schools to educate, to be responsive, to be efficient, and to innovate.}\]

\((\text{Moe, 2008})\)
Improvement Incentives  (Hatfield–Kojima–Narita, 2012)

Definition

A mechanism respects improvements of school quality if when students rank school \( c \) higher, \( c \) obtains a “better” set of students.
Definition

A mechanism respects improvements of school quality if when students rank school $c$ higher, $c$ obtains a “better” set of students.

Bad News

- No stable mechanism (e.g., SOSM) respects improvements of school quality.
- No Pareto efficient mechanism (e.g., Boston, TTC) respects improvements of school quality.
- These negative results are quite general.
Improvement Incentives (Hatfield–Kojima–Narita, 2012)

Definition
A mechanism **approximately respects improvements of school quality** if for “almost all” preference profiles, no school is better off when students demote it in their rankings.

Good News
- Any stable mechanism (e.g., SOSM) approximately respects improvements of school quality.
- The Boston and TTC mechanisms do not approximately respect improvements of school quality.
- “Large market” results in tradition of Immorlica–Mahdian (2005) and Kojima–Pathak (2008) (see also Azevedo–Leshno (2012)).
Improvement Incentives (Hatfield–Kojima–Narita, 2012)

SOSM incentivizes improvement; Boston, TTC do not.

(More generally, market designers need to consider the impact of design on agents’ long-term incentives.)
School choice advocates emphasize the effect of school choice on schools’ incentives to improve themselves.
“No choice in school choice?” (Calsamiglia–Miralles, 2012)

School choice advocates emphasize the effect of school choice on schools’ incentives to improve themselves.

Even more, they emphasize the fact that school choice programs should actually enable choice.

*School Choice is... a common sense idea that gives all parents the power and freedom to choose their child’s education [...].*

*(The Friedman Foundation for Educational Choice)*
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School choice advocates emphasize the effect of school choice on schools’ incentives to improve themselves.

Even more, they emphasize the fact that school choice programs should actually enable choice.

School Choice is... a common sense idea that gives all parents the power and freedom to choose their child’s education, while encouraging healthy competition among schools [...] .

(The Friedman Foundation for Educational Choice)
“No choice in school choice?” (Calsamiglia–Miralles, 2012)

Definition

A mechanism does not respect the spirit of school choice if it always assigns students to their “neighborhood schools.”
“No choice in school choice?” (Calsamiglia–Miralles, 2012)

Definition

A mechanism does not respect the spirit of school choice if it always assigns students to their “neighborhood schools.”

Bad News

- In a large market with binary, “neighborhood” priority and agreement as to the worst school, neither SOSM nor Boston respects the spirit of school choice.
- Natural “fixes” involve favoring already-advantaged students.
"No choice in school choice?" (Calsamiglia–Miralles, 2012)

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A mechanism does not respect the spirit of school choice if it always assigns students to their "neighborhood schools."

Bad News
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- Natural "fixes" involve favoring already-advantaged students.

Slightly Better News
- TTC does respect the spirit of school choice (but does not help students who live in the bad neighborhood).
Pessimistic conclusion? No!

School choice market design has enabled access to good schools in \( \{ \text{Boston, New York, Chicago, \ldots} \} \).
Pessimistic conclusion? No!

1. School choice market design has enabled access to good schools in \{\text{Boston, New York, Chicago, \ldots}\}.

2. Work is ongoing in \{\text{New Orleans, San Francisco(\ldots), \ldots}\}. 
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3. We have learned a tremendous amount about priority-based allocation (also useful in other applications (coming up next)).
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2. Work is ongoing in \{New Orleans, San Francisco(?), \ldots\}.

3. We have learned a tremendous amount about priority-based allocation (also useful in other applications (coming up next)).

And as to getting students out of especially bad neighborhoods:

- Tomorrow, we will incorporate affirmative action.
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To increase officer retention, the Army recently introduced a “branch-of-choice” program, in which cadets may “bid” for priority.

- This system is a sort of “simplified auction” (technically fascinating, from matching-theoretic perspective).
- Today, we will focus on the inequality/diversity issues.
Diversity Concerns  (Lim, 2009)

- Military leadership is demographically homogeneous: In 2006, only about 16% of officers were African American or Hispanic.

- Scarcity of minorities in combat arms branches is a barrier to improving diversity in the senior ranks.

- While 58% of white cadets’ submitted first choices were in combat arms, only 31% of African American cadets’ were.
On the one hand, minority cadets could truly prefer different career fields than white cadets. In this case, policy should focus on ways to make combat career fields more appealing to minorities. On the other hand, minorities may not really prefer support career fields but rather may reason that they lack the OML to get a more competitive career field [...] and may opt for their most-preferred Combat Support or Combat Service Support career field [...].
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Sound familiar?
Design Sketch  \textit{(Sönmez, 2011)}

Using the (full) cadet-optimal stable mechanism,\textsuperscript{1} can solve these problems—and more!

- Stable, strategy-proof, and improvement-respecting.

\textsuperscript{1}with well-chosen priority structure
Holdout in the Assembly of Complements

- Ten people own (privately valued) homes

\[
\begin{array}{ccccccc}
& & & & & & \\
\end{array}
\]

You want to buy their land and build a mall (worth 90). All you know is that their values are uniformly distributed in \{1, \ldots, 10\} (expected total value 55). What should you do?

- Take-it-or-leave-it offers of 1, \ldots, 10 (total 55)?
  \[p(\text{sale}) = 10 - 10 = .0000000001\]

- Take-it-or-leave-it offers of 8 (total 80)?
  \[p(\text{sale}) = \frac{8}{10} < .11\]

- Self-assessment: ask owners to reveal their values?

- Eminent domain: take homes and pay each owner 1 (total 10)
Ten people own (privately valued) homes

\[
\begin{array}{cccccccccc}
2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 1 \\
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Holdout in the Assembly of Complements

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  ![Illustration of ten homes](image)

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- What should you do??
  - Take-it-or-leave-it offers of 1, \ldots, 10 (total 55)?
    - \( p(\text{sale}) = 10^{-10} = 0.0000000001 \)
  - Take-it-or-leave-it offers of 8 (total 80)?
    - \( p(\text{sale}) = \left( \frac{8}{10} \right)^{10} < 0.11 \)
  - Self-assessment: ask owners to reveal their values?
  - Eminent domain: take homes and pay each owner 1 (total 10)?
Basic Model

- Buyer has (private) value $b$ for aggregate plot.

- Each seller $i$ has (private) value $v_i$ for her land.

- Each seller has expected share of total value $s_i$.
  - can be entirely exogenous or determined by buyer
  - $s_i$ close to $v_i/(\sum_j v_j)$ $\implies$ better property rights

- A *mechanism* is a transaction procedure.
Design Goals: Ideal

1. Fully Efficient: mechanism captures all gains from trade
   - Sale $\iff b \geq \sum_i v_i \equiv V$

2. Individually Rational: no seller sells for less than value
   - Sale $\implies$ each seller $i$ receives at least $v_i$

3. Budget-Balanced
   - No transfers to/from the market-maker

1. Straightforward for Sellers: truthful play dominant (for sellers)
2. Bilaterally Efficient: as efficient as bilateral trade
   - Sale $\iff o^*(b) \geq V$
3. Partial Individual Rationality
   - Approximate IR: seller $i$ receives at least $\frac{s_i(V - v_i)}{1 - s_i}$
   - Collective IR: community not forced to sell for less than $V$
4. Self-financing
   - No transfers from the market-maker

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Concordance among Holdouts (K.–Weyl, 2012)

1. Introduce holdout as a market design problem
   - Goals – straightforwardness, bilateral efficiency, approximate IR

2. Propose solution approach
   - “Concordance” – divide profits according to \( ex \ ante \) shares \( s_i \)

3. Investigate when competition offsets complementarity
   - Combinatorial holdout – clusters and repacking
In Concordance Mechanisms:

1. Sellers $i$ divide offer $o$ into previously-specified shares $s_i o$.
2. Each seller pays a Pigouvian tax for externalities.
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Properties

1. Collective rationality and approximate individual rationality
2. Bilateral efficiency and asymptotic efficiency *under truthfulness*
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- Problem: choice of collective decision-making procedure
Concordance among Holdouts  (K.–Weyl, 2012)

Problem: choice of collective decision-making procedure
Problem: **choice of collective decision-making procedure**

- VCG – vulnerable to collusion, not budget-balanced
- expected externality, voting – require distributional information
- legal recourse – buyers can exploit coercive power
- quadratic vote buying (Weyl, in preparation) – . . . ?
Overview

Today

- The Market Design Approach
- Design of School Choice Programs
- Cadet–Branch Matching; Eminent Domain

Tomorrow

- Design of Affirmative Action Mechanisms
Overview

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What is Market Design?

Application of economic principles and game theory to the design (or re-design) of market institutions.
What is Market Design?

1. Economic Engineering
   e.g., improving incentives; “leveling the playing field”

2. Working Around Impossibility Results
   e.g., no-trade theorems; nonexistence results

3. Working Within Existing Conditions (where possible/necessary)
   e.g., existing policy goals

4. Organizing Market Function
   e.g., strategy-proof mechanisms $\rightarrow$ accurate data
Some Key Concepts

1. Strategy-proofness (vs. Manipulability)
   - essential for ensuring simplicity; not always achievable

2. Market Thickness
   - success requires *participation*

3. Evaluation Criteria
   - vary from setting to setting; often depend on policy goals

4. Flexibility
   - often crucial for market organizers
See you tomorrow!

Today
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