Appendix to *Concordance among Holdouts*

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I Cournot

This appendix reviews and extends Cournot's classic results, connecting them to the design of mechanisms for multilateral trade. Little in this section is "new," as it largely seeks to unify the proofs of disparate results in the literature. However, we believe that this simple united treatment helps elucidate the connections among a variety of issues previously discussed in isolation and, emphasizes the importance of the holdout problem.

I.A Cournot's Theorem and Bergstrom's Corollary

We begin with the classic theorem of Cournot (1838): Increasing the number of competing, symmetric, constant-marginal-cost firms leads to efficiency.

Cournot's Theorem. *Suppose that demand is finite when price is at marginal cost. Then, as the number of perfectly substitutable, quantity competitors grows large, price approaches the (common) constant marginal cost.*

As the number of firms increases, each has little impact on price. Alternatively because each is a small part of the market, each cares little about the impact it has on price because most of this impact is borne by other firms.

More formally, suppose there are N firms with constant marginal cost of production c. Market inverse demand is $P(Q)$, where Q is the aggregate production. The production of firm i is q_i; let $Q_i \equiv Q - q_i$. Firm i's revenue is $P\left(q_i + Q_i\right) q_i$ and the firm takes Q_i as given. Thus firm i's marginal revenue is $P(Q) + q_i P'(Q) = P + \frac{Q}{N} P' = \frac{N-1}{N} P + \frac{1}{N} \mathrm{MR}_M$ where MR_M is the marginal revenue of a monopolist facing the same demand curve. It follows that, as N grows large, the Cournot oligopolist's marginal revenue approaches that of a competitive firm, namely price, and therefore her price converges to marginal cost.[1] This is depicted in Figure 1, for the inverse demand $P(Q) = 1 - Q$.

[1] A general proof of this fact is given by Frank (1965).

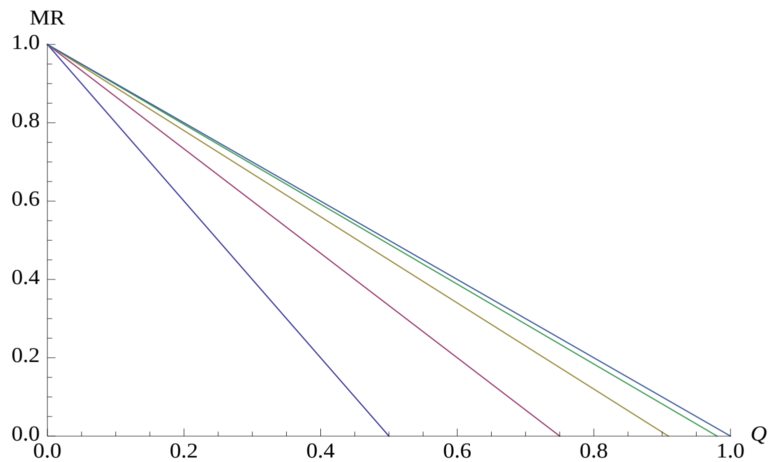

Figure 3: Marginal revenue as a function of quantity, given market inverse demand $P(Q) = 1 - Q$, for various numbers of competitors. The bottom curve is monopoly, the top inverse demand and competition increases as one moves up the curves.

As Sonnenschein (1968) pointed out, collaboration is the dual of quantity competition. Bergstrom (1978) used this duality to prove a natural corollary to Cournot's Theorem for the collaboration problem.

Bergstrom's Corollary. *Suppose that demand has the property that elasticity increases in price or that price is finite when quantity is* 0. *Then as the number of collaborating firms monopolizing components of a good (with fixed demand) grows large, the quantity sold approaches* 0.

An increase in the number of collaborators is harmful, as each fails to internalize the harm that an increase in her price causes other firms' sales. This point was made by Spengler (1950), who argued that a monopolistic upstream (coal) company selling to a downstream (steel) monopolist would lead to consumers facing two mark-ups or "double marginalization." More generally, as the number of collaborators grows large, the impact of any one on the quantity purchased becomes small. Equivalently, because the firm earns a small part of the total mark-up, she cares little about the impact an increase in her price has on sales. Therefore each firm eventually demands an arbitrarily high price, eliminating all sales.

Letting $Q(P)$ be the demand for the final good as a function of the total price, one can now follow the above reasoning in perfect parallel. Let p_i be the price of firm i and let $P_i \equiv P - p_i$. Let the total marginal cost of producing the product c be fixed and assume each firm bears an equal share of this cost. Profits are then $Q\left(P_i + p_i\right)\left(p_i - \frac{c}{N}\right)$. Optimal pricing requires $Q + Q'\left(p_i - \frac{c}{N}\right) = 0$ or equivalently $Q + Q'\frac{P-c}{N} = 0$. Thus, as N becomes large, Q must converge to 0.

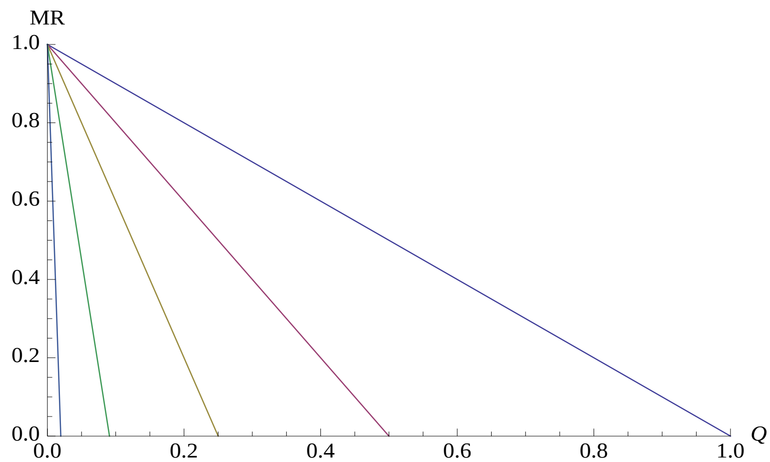

Figure 4: Marginal revenue as a function of quantity, given market inverse demand $P(Q) = 1-Q$, for various numbers of collaborators. The top curve is the inverse demand, the second from the top the monopolist and from there on down the number of collaborators increases.

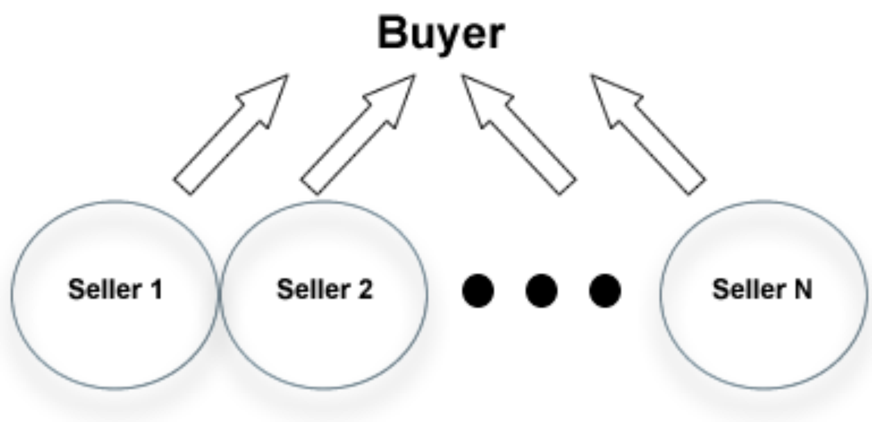

Figure 5: Perfectly substitutable procurement: many sellers compete to offer identical goods.

Another way to see this is in terms of marginal revenue curves. The revenue of a perfectly complementary monopolist is $(P(Q) - P_i)\,Q$ where each monopolist can now choose Q as the goods are perfect complements. Marginal revenue is $P(Q) - P_i + QP'(Q) = \frac{P}{N} + QP'$. Because firm i only bears $\frac{1}{N}$ of the total marginal cost, we must multiply this by N to obtain the effective industry marginal cost: $P + NQP' = N\text{MR} - (N-1)P$. Thus rather than marginal revenue approaching price, it becomes ever more distant approaching the horizontal axis, as shown in Figure 2.

I.B Multilateral Trade

Cournot's problems have natural analogs in the context of multilateral trade.These are models in which the traders on one side of a bilateral bargain (Myerson and Satterthwaite (1981)) grow in number, and are viewed as either perfect substitutes or perfect complements by traders on the other side. For the problem of optimal mechanism design, the side on which growth takes place is irrelevant.[2]

For simplicity, therefore, we consider the following concrete situation, paralleling our first application in Section II.A (below). A buyer wants to purchase a plot of land, and has a value associated with purchasing it that is her private information and is distributed according to some continuous density f on $\left[\underline{b}, \overline{b}\right]$. We can consider two possibilities for sellers: One is that there may be many different plots of land, each with a single owner, that are identical to the buyer. This situation, corresponding to Cournot's model of competition, is pictured in Figure 5. Another is that there may be many owners of pieces of a single plot of land, which is the only plot the buyer is interested in. This setting, corresponding to Cournot's model of collaboration, is pictured in Figure 6.

I.B.1 Perfect substitutes

First consider perfect substitutes. Each of N potential sellers has a value for her land that is private information. This value v_i is distributed independently and identically according to some density g on $[\underline{v}, \overline{v}]$. Let $\nu \equiv E[v]$, and assume that $\nu < \underline{b} < \overline{b} < \overline{v}$, so that—regardless of the buyer's value—trade is on average efficient.

To maximize welfare, a social planner naturally wants to encourage purchase of the plot of seller i in states where i's valuation is below some cutoff value $\tilde{v}$, which possibly depends on other information available to the planner has. The seller holds property rights to the plot and therefore can choose whether to sell. To persuade the seller to agree whenever she has a value below $\tilde{v}$, the social planner must make her a take-it-or-leave-it offer of $\tilde{v}$, as the classic envelope theorem (Bulow and Roberts, 1989; Milgrom, 2004) demonstrates. Thus if the social planner wants to purchase seller i's land with a probability of q_i, she must on average pay $G^{-1}(q_i)q_i$ to seller i. To parallel more closely the Cournot problem, we let $P(q_i) \equiv G^{-1}(q_i)$. Therefore, the marginal cost of buying with a slightly higher probability from seller i is $\text{MC}_M\left(q_i\right) \equiv P(q_i) + q_i P'(q_i)$, labeled with an M subscript as this is the marginal cost of purchasing from a monopolist (Bulow and Roberts, 1989).

[2]In the substitutes case, we may have many equivalent-to-the-seller participants in an auction, or many equivalent-to-the-buyers competing sellers. In the complements case, we may have buyers who can only complete a purchase if all buy their components, or many sellers facings a buyer who views their various goods as perfect complements.

Now note that in order to purchase some plot, the social planner need only buy from any given seller infrequently. A simple way to see this is to imagine that the social planner randomly selects a sequence for the sellers, offering each a take-it-or-leave-it offer $P(\tilde{q})$ which, of course, has a chance $\tilde{q}$ of succeeding. If it succeeds, then the planner now asks the buyer whether he would like to purchase the plot for $P(Q)$ and if so carries out the transaction, otherwise ending the process. If she fails, then she makes the same offer to the next seller, and so forth. Clearly this procedure requires no infusion of cash from the market maker and is individually rational for sellers.

The chance that she purchases at least one plot is then $Q \equiv 1 - (1 - \tilde{q})^N$ and the chance that she purchases from any given seller is $\frac{Q}{N}$. The cost to society of buying some plot with probability Q is then $QP\left(\tilde{q}[Q, N]\right)$ and thus the social marginal cost of the purchase is $\tilde{q}(Q, N) = 1 - (1 - Q)^{\frac{1}{N}}$.

Thus *just as in the Cournot problem* the marginal cost of purchasing with probability Q converges towards "price" (the seller's value) as a weight $\frac{N-1}{N}$ is on price and $\frac{1}{N}$ is on monopolistic marginal cost. Furthermore because P converges to $\underline{v}$ the probability that the buyer is willing to make the purchase approaches 1. There are only four small differences:

1. The price curve also changes with N, converging towards $\underline{v}$.

2. It is now the *marginal cost* to the social planner of the purchase, rather than the *marginal revenue to the sellers* that converges towards its price curve.

3. The quantity represents the probability, rather than amount, of purchase.

4. If better mechanisms are used, then convergence may be somewhat faster than the bound given here. For example, an auction may be used to keep the (conditional) sales from any particular seller below $\tilde{q}(Q)$ most of the time. Figure 7 shows the lowest possible actual marginal cost curves as a function of Q for various N for $P(Q) = 1 - Q$.

Corollary 1. *As the number of perfectly substitutable sellers grows large, there is a self-financing, individually-rational mechanism achieving Q arbitrarily close to* 1.

Note that key to this result is *not* the fact that, as N grows large, the social planner is certain that a trade should take place. Rather, it is the fact that she need not buy from any given seller very often; this means that any given seller has little leverage over price. Just as Cournot competition erodes the incentive to reduce production by making each firm a small part of the market, the presence of a large number of sellers erodes the benefits of refusing to sell.

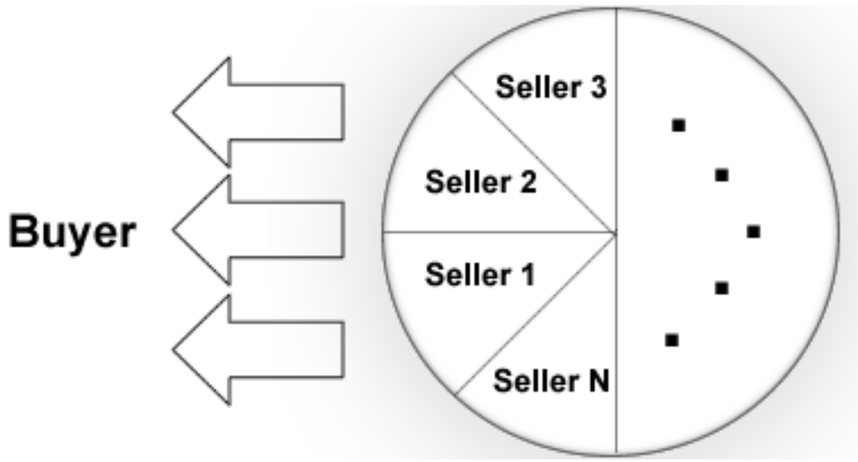

Figure 6: Perfect complements procurement: sellers must all agree to a sale.

Of course this is certainly not the only, in fact not even nearly the most efficient, mechanism.[3] However, even this simple (and suboptimal) mechanism performs extremely well when there are many competitors. Consequently, we see that if competition is intense, then design may not be paramount.

I.B.2 Perfect complements

Now consider the opposite, perfect complements case in which there is only a single desired plot of land, but it is split into many pieces each owned by a different individual. Each individual has a value for his or her land that is identically and independently distributed on $\left[\frac{\underline{v}}{N}, \frac{\overline{v}}{N}\right]$ according to the distribution $g_i(v_i) = g(Nv_i)$; that is $v_i = \frac{V_i}{N}$ where V_i is a draw from g. Thus, by the same argument as above, the cost of purchasing from seller i with probability q_i is $\frac{P(q_i)q_i}{N}$.

Shavell (2007) proposes, as a model real-world negotiations, a mechanism where the social planner or a buyer offers a given price and a sale is made only if all sellers agree. Grossman et al. (2010) propose a similar mechanism where the (potentially competing) buyer(s) offer a price which is allocated among sellers according to some exogenous share rule and then, again, the sale goes forward only if all sellers agree. These essentially similar (see Section IV.E) mechanisms provide a simple demonstration of the logic of the holdout problem. For simplicity, we discuss Shavell's mechanism to illustrate the problem.

Under Shavell's mechanism, if an offer of $P(\tilde{q})$ is made, the probability that each seller

[3] A familiar alternative is the *double auction*, in which participants bid their values, beneficial trades are carried out, and price is determined by an averaging rule. The same logic presented above shows that double auction outcomes quickly converge towards efficiency (Satterthwaite and Williams, 1989; Rustichini et al., 1994) under conditions much more general than those considered here (Cripps and Swinkels, 2006).

consents to the sale is $\tilde{q}$. Thus if the social planner would like, possibly conditional on some information from the bidder, to achieve a sale with probability Q she will need to choose $\tilde{q}$ so that $\tilde{q}^N = Q$ or $\tilde{q}(Q) = Q^{\frac{1}{N}}$. This costs the planner $P\left(Q^{\frac{1}{N}}\right)Q$. For any Q, as N grows large, this approaches $Q\overline{v}$, a price which (by assumption) no buyer would be willing to pay. In order to persuade every seller to agree to a sale through a uniform offer, the buyer must offer the highest of any seller values. If there are many sellers, then this value will usually be too high to allow sale. Therefore, in the presence of many perfectly complementary sellers, the probability that even a profitable sale can take place is very small.

But perhaps this is too pessimistic. After all, Shavell's mechanism seems a bit naïve: why make the same offer to every seller? Why not try to elicit from sellers their private valuations, using the fact that low-valuation sellers benefit more from sales? The problem is that no individual seller is likely to break a deal; hence, each seller has tremendous leverage.

For illustration, suppose one wanted to achieve a sale with some reasonable probability just in the 50% of cases when $V \equiv \sum_i v_i < \nu$, in which trade is beneficial irrespective of the buyer's value. Let $V_i \equiv V - v_i$. Then the chance that any particular v_i matters in deciding whether one is in the set of cases where $V < \nu$, the chance that i is *pivotal*, is very small. Precisely, it is the chance that $\underline{v} < N(\nu - V_i) < \overline{v}$. V_i is the sum of $N - 1$ i.i.d. random variables and thus has variance $\frac{1}{N}$; therefore NV_i has variance N and the probability of i being pivotal dies off quickly with N. That is, no seller is likely to scupper the whole deal. Therefore, for large N it is nearly always the case that whether a sale will occur does not depend on v_i. Moreover, for most values of other sellers, seller i must be non-pivotal and thus must have q_i near 1. Thus every seller will try to demand the maximum credible value for her land $(\frac{\overline{v}}{N})$ most of the time.

This reasoning, outlined by Rob (1989) and refined by Mailath and Postelwaite (1990) for public goods games, is another elegant corollary of Cournot's Theorem, through the Bergstrom Corollary. Figure 7 shows the total marginal cost curves for the least-cost strategy, described in the formal proof below, for the inverse demand $P(Q) = 1 - Q$.

Mailath–Postelwaite–Rob Corollary. *As the number of perfectly complementary sellers grows large, the amount of trade under any self-financing, individually-rational mechanism must vanish.*

This corollary highlights a fundamental holdout challenge: a large number of perfectly complementary sellers eliminates all gains from trade. This is despite the fact that (by the Central Limit Theorem) the aggregate community value converges to $\nu < \underline{b}$ and thus with many sellers *it is always efficient for a sale to take place*! The fact that it is asymptotically certain that trade is beneficial does not render trade feasible.

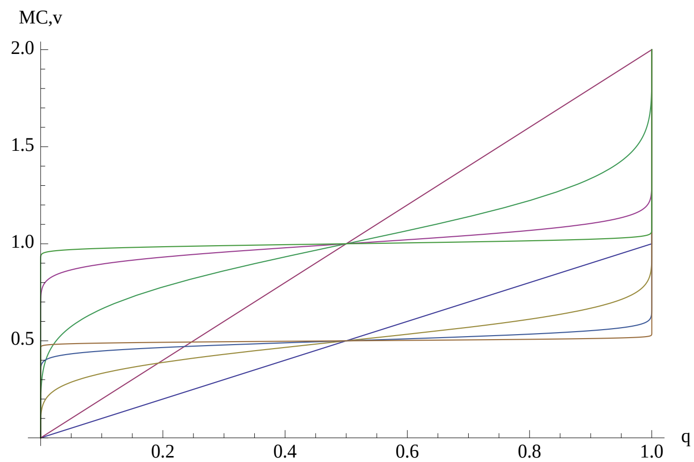

Figure 7: Marginal cost and value curves as a function of quantity for perfect complements procurement for a valuation distribution uniform on $[0, 1]$ and various numbers of sellers. Value approaches the average value of $\frac{1}{2}$, but marginal cost converges to the maximum value of 1, for every probability of sale q as the number of sellers grows.

Thus, outcomes in markets with many complementary sellers depend crucially on market design. We label this the *holdout problem.*[4] It is to solving this problem that we devote our paper.

Proof of the Mailath–Postelwaite–Rob Corollary. We adapt the argument of Mailath and Postelwaite (1990): The cheapest method of achieving trade with a given probability Q is clearly to purchase in the cases for which total marginal cost of purchase is lowest, so long as marginal cost is increasing in value (which we have assumed). This aggregate marginal cost of the plot is $\text{MC}(\mathbf{v}) \equiv \sum_i \text{MC}(v_i)$. This aggregate marginal cost is just a random variable with some distribution $\overline{G}(x) \equiv \text{Prob}\left[\sum_i \text{MC}_i(v_i) < x\right]$ and corresponding density $\overline{g}$.

The aggregate marginal cost of selling with probability Q is therefore

$$\overline{\text{MC}}(Q) \equiv \overline{\text{MC}}\left(\overline{G}^{-1}[Q]\right).$$

Note that $\overline{\text{MC}}$ has the same distribution as $\frac{1}{N}\sum_i \text{MC}_M(V_i)$ where V_i are drawn i.i.d. from G. Clearly $E\left(\text{MC}\right) = \overline{v}$, as the average value of marginal cost is just average cost and the average cost of purchasing a plot with probability 1 is $\frac{P(1)1}{1} = \overline{v}$. Let $\sigma^2 \equiv \text{Var}\left[\text{MC}\right]$.

[4]Others have given other definitions to the holdout problem (Miceli and Segerson, 2007; Menezes and Pitchford, 2004) (such as delay in bargaining). However, we view the basic holdout problem we emphasize as the fundamental source of delay in any particular bargaining game, the same way it prevents sales in the setting of Shavell (2007).

By Chebyshev's inequality,

$$P\left[\overline{\text{MC}} < \kappa - k\frac{\sigma}{\sqrt{N}}\right] < \frac{1}{k^2}.$$

Equivalently, for any $k > 0$, we have $\overline{\text{MC}}\left(\frac{1}{k^2}\right) > \kappa - k\frac{\sigma}{\sqrt{N}}$. Thus every point on $\overline{\text{MC}}$ becomes arbitrarily close to $\overline{v}$, just as every point on the the marginal revenue curve with perfectly complementary monopolies approaches 0. It follows that the marginal cost of purchasing land with *any probability* converges uniformly to the most expensive possible price for the land. This makes even *breaking even* impossible for the social planner (for any fixed level of trade Q) as N grows large, and thus asymptotically eliminates all opportunities for trade.

II Applications

This appendix elaborates on the abbreviated discussion of applications presented in the main text of our paper.

II.A Land assembly (takings and collective property)

Perhaps the best-known and most-studied instance of the holdout problem is land assembly. It is often the case that a large plot of land, owned in pieces by many individuals, is valuable to prospective buyers in only its entirety.[5]

This extreme complementary may arise for physical reasons. When a government attempts to build a highway, a military base, or another public work, there is a minimum scale required to make the project possible; this minimum scale necessarily requires a minimum size of the land plot on which the project may be built, making all sub-plots nearly perfectly complementary. The resulting holdout problem is often offered (Merrill, 1986; Posner, 2005) as a rationale for the policy of "eminent domain" or property takings written into the Fifth Amendment to the US Constitution, whereby governments may take "private property ... for public use," but only after "just compensation" has been paid.

This procedure typically involves the (local) government asking for an assessment of land values by a real estate expert. Compensation tends to be far below the minimum price at which most residents would be willing to sell, as

- the real estate experts hired work for the government,

[5]While perfect complementarity may be rare in practice, it suffices for our purposes that the buyer's value decline precipitously if any part of the plot is left out of the package.

- eminent domain procedures typically follow a period in which land has been declared blighted, and
- legal recourse for takees is increasingly limited, following a recent supreme court decision (*Kelo v. City of New London, Connecticut*, 2005).

The landmark *Kelo* decision resolved much of the ambiguity in past case law, dramatically expanding the number of, as well as controversy surrounding, takings. The number of takings cases expanded from just over 2000 per year in the 1998-2002 period to nearly 6000 in the year following the decision (Berliner, 2006). To combat this, twenty-two states have undertaken substantial reforms of their takings rule (Castle Coalition, 2009). These have ranged from laws and proposed laws in New Hampshire, Missouri and Tennessee that require payments of as much as three times the appraised value in compensation (Morton, 2006), legislation that may well make many efficient land assemblies unattractive to local authorities. Many more, including Florida, Georgia, Louisiana, Michigan, New Hampshire and South Carolina, have effectively banned whole classes of takings (Wolf, 2008). This has directly revived the holdout problem by forcing prospective land assemblers to revert to decentralized multilateral bargaining. Clearly both the basic eminent domain system and the blunt reactive remedies instituted by states to correct it are deeply inefficient.

Holdout problems may also emerge when land is owned collectively. For example, the Mexican *ejido* system requires the consent of all member of a community for any land in that community to be sold or a suitable alternative mechanism for a collective decision on the sale of the land. The system originally encompassed more than one hundred million hectares, more than half of the country's land area and nearly all arable land (Cevallos, 2008). Schmidt and Gruben (1992) argues that the resulting, legally-engineered holdout problem is a primary reason for the "national agricultural system dominated by uneconomically small, undercapitalized farms," and the cost of land transactions in rural México. Because of the difficulties of land transfer under ejido, seventy million hectares still remain publicly owned, with export-oriented agriculture overwhelmingly concentrated on the small fraction that has been transferred to commercial producers (Cevallos, 2008).[6]

Holdout may also result from social, cultural and political factors. A classic example is in Rio de Janeiro, where more than two million poor and lower middle class *Cariocas* reside in *favelas*, or slums, on the beautiful hillsides overlooking the city and bay. The land in these favelas is not officially owned by its occupants, but an informal system of property, respected by residents and the state, has arisen. Because the favelas are plagued with violence, crime

[6] Many commentators, such as Johnson and Fitzgerald (2003), believe that inefficient management of Mexican farm land is a major cause of migration for agricultural labor to the United States.

and drug trafficking, and because land titles remain informal, virtually no development has taken place on this choice real estate. A settlement that would make the neighborhoods livable for the Carioca elite would require removing the entirety of large slums, a proposition that is politically infeasible without the consent of all residents.

The costs of this inefficient allocation are astonishing. In Rio alone 1.2 million residents live in favelas where two-bedroom homes sell for as little as $1,400. Meanwhile, a comparable house just outside the favela sells for more than $16,000 (Rich, 2001), and a single square meter in an apartment in the posh Ipanema neighborhood that is only a ten minute walk from favela *Rochina* costs upwards of $6,000 (Denyer, 2008). Suppose that a typical two bedroom house is on a plot of 50 square meters and that the value of a square meter of land is approximately that of a square meter of apartment space—this likely understates the value given that typically apartment buildings are many stories. Then the land on which a typical two-bedroom home sits might be worth as much as $300,000 in its most efficient use. Supposing that typical value is only half of this and that a two-bedroom house typically has four residents, there would be $45 billion in gains from reallocating the land to its most efficient use.

This problem stretches far beyond Brazil, throughout Latin America and in many other developing countries in Asia and Africa. In India, development-oriented land assembly projects alone displaced more than 20 million residents between independence and 1991 (actionaid, n.d.). Perhaps the most famous international example of land assembly holdout occured in Japan: holdouts forced Narita airport to relocate (by 800 meters) and shorten (by 320 meters) the second runway it opened in 2002 (Shimizu, 2005).

Broader empirical data would be extremely useful in quantifying the size of the land assembly problem. However a simple heuristic calculation is instructive. Suppose, conservatively that formal takings cases in the United States are one fifth of all land assembly problems and that the typical such problem is a million dollars in size. Suppose that the United States constitutes one fifth of annual land assembly problems globally—also conservative given the epic scale of such challenges in developing countries with weak institutions. Then global land assembly problems are on the order of hundreds of billions of dollars.

While our model clearly fits land assembly, it is instructive to consider how some of ancillary concepts defined in Section III of the main text fit into the picture. At a basic level our assumption that sellers value only their own plots is motivated by the fact that it is unlikely that any seller has an idiosyncratic value for any plot of land other than her.

Similarly, shares are naturally defined in the setting of land assembly. Clearly, the process of assessing exact land values (as in eminent domain) is at best biased and highly speculative. Even an honest assessment of fair market value would by its nature fail to account for the fact that owners who choose to stay in their home clearly reveal that they value that home more than the amount they would be able to sell it for. However, as Grossman et al. (2010) argue, the *relative valuations* assessed for various plots of land are likely somewhat more accurate and, depending on the sales procedure, may be less biased. We agree with Grossman et al. (2010) that it therefore seems reasonable to assume that a share of the total plot can be assigned to each individual land-owner through a fair and objective procedure.[7]

Many of the criteria we advocated in Section III of the text are clearly desirable in the context of land assembly. Most sellers of land, especially in the developing world, are not professional developers and are likely financially unsophisticated, if not uneducated and impoverished. It seems unlikely that any systems that would force them to make complex calculations could succeed. The direction of recent Supreme Court rulings strongly indicates that the constitutional requirement that land be taken only for "public use" is properly interpreted as necessitating only an unambiguous demonstration of the social benefits, read efficiency, of the project (*Hawaii Housing Authority v. Midkiff*, 1984; *Kelo v. City of New London, Connecticut*, 2005). In the landmark decision that laid the ground for recent case law, the court (*Berman v. Parker*, 1954) writes that eminent should be drive by the "public interest" as "declared in terms well-nigh conclusive" by the "legislature." In his classic analysis of takings, Michelman (1967) argued that eminent domain is intended to allow "enterprisers" to assemble land when the benefits of the project exceed the costs to "losers."

The preservation of property rights (i.e. individual rationality) is a crucial objective of land assembly regimes, as embodied in the requirement of just compensation for loss of property. However in many contexts the limited individual rationality notions defined in Section III of the text satisfy these goals as well as (if not better than) strict protection of individual rationality. For example, protecting collective rationality seems particularly appropriate when the plot is collectively owned by a community of individuals.

On the other hand, the Anglo-Saxon tradition of thought on takings (Locke, 1689; Dana and Merrill, 2002) emphasizes the inviolability of individual property rights. However, an alternate continental tradition of thought (Pufendorf, 1682; de Vattel, 1758; Van Bynkershoek, 1737), exemplified by Grotius (1646) (who is thought to have first coined the phrase "eminent domain") also substantially influenced provisions for just compensation, even in the United States. Rather than the inviolability of property and the necessity of consent, this tradition emphasized the importance of a fair sharing among citizens of the burdens of social projects. This view resonates with contemporary legal scholarship on just compensation. For example, Ellickson (1973) argued that just compensation aims at "a community consensus

[7]The notion of applying shares—typically used in corporate acquisitions—to takings was (as far as we know) independently and simultaneously arrived at in both our work and that of Grossman et al. (2010).

on the severity of the harm inflicted." Our notion of approximate individual rationality is a direct formalization of this idea: each individual is entitled not to her idiosyncratic valuation but to a fair and objectively determined share of the community's subjective valuation of her land.

II.B Corporate acquisitions and other examples

When one individual or corporation wishes to gain a controlling share in a publicly-owned company, the laws of most developed countries requires that they make a bid for all shares of the firm (Kirchmaier et al., 2009). These regulations are designed to protect minority shareholders' interests from the potentially conflicting interests of another profit-maximizing firm with a majority stake. Of course they have the unfortunate side effect of creating a holdout problem, as all disparate shareholders would, in principle, need to consent to a sale. This is addressed differently in various jurisdictions, as discussed below.

To see how corporate acquisitions fit the model of our paper, assume that the ownership of a corporation is divided into M shares, where M is thought to be a very large number (on the order of hundreds of millions). There are N investors $i = 1, \ldots N$, where again N is large, on the order of millions. Each investor gains some value from owning her shares; we assume this is quasi-linear in current cash-on-hand:[8] $U_i(m_i) = u_i(m_i) - pm_i$, where m_i is the number of shares owned by investor i and p is the market price for the shares. We rule out short-selling by assuming that investors must have $m_i \geq 0$. I assume all u_i are twice-differentiable and strictly concave, because the more shares of the firm a risk-averse individual holds, the more exposed she is to the risk associated with a decline in that firms' value.

In the absence of an acquisition, some market price for the shares, $p^\star$, will prevail.[9] Let $v_i \equiv u_i(m_i^\star)$, where $m_i^\star$ maximizes $u_i(m_i) - p^\star m_i$. This setting again has sellers that are only interested in their own "plots" as, in the absence of an acquisition, an open market for shares of the company will prevail and the value to an individual of receiving a share is just its market price. Regardless of the allocation of shares, equilibrium will be reached and each individual will hold her "plot," $m_i^\star$ of the identical shares. The private-information component of her value from this is $v_i - m_i^\star p^\star$, her surplus from owning $m_i^\star$.

Again we have a natural and commonly used criterion for shares. If we believe that all public information on utility functions is captured in the number of shares an individual

[8] This just rules out the individual stock being a large fraction of an investor's total wealth.

[9] Note that the equilibrium price is (nearly) unique, because utility is quasi-linear. Of course, given the finite number of investors and the potential of asymmetric information (Kyle, 1985), this may involve a bid-ask spread, but presumably this is fairly small for a liquid market such as that for a publicly owned firm.

chooses to purchase at a given price, then the natural share criterion is $s_i = \frac{m_i^\star}{M}$. Note that by the assumption that $p^\star$ is an equilibrium price, $\sum_i m_i^\star = M$, so $\sum_i s_i = 1$. Furthermore shares are observable: presuming all individuals have the same beliefs about the chance that acquisition occurs and have common risk-preferences, the number of shares purchased at ex-ante prices are identical to those purchased at the equilibrium price conditional on no sale.

Because it is nearly impossible for a prospective buyer to purchase all shares directly—and legal requirements typically necessitate a complete purchase—various states and countries have different rules for determining whether an offer is accepted. Nearly all of these involve obtaining the assent of some fraction of current shares, although some involve additional protections, such as requiring the offer to exceed recent past as well as present market prices (Guriev et al., 2004). In Europe, acquisition systems typically have higher thresholds, as high as 90-95%, and are typically implemented through a system where a buyer must make an offer that is accepted by the required fraction of shareholders. Other shareholders are then "squeezed-out" at the price accepted by the majority under securities law (Croft and Donker, 2006). In the United States, on the other hand, after an offer is tendered, shareholders typically conduct a formal vote on whether the offer should be accepted, with the threshold determined on a state-by-state basis (Armour and Skeel, 2007). The efficiency of these systems, especially those with high thresholds, can be very low (Bukart and Panuzi, 2003). Meanwhile, these systems provide poor property rights protections (der Elst and den Steen, 2006). Especially given the large size of corporate acquisitions globally (which, according to Dealogic, amounted to $972 billion or 5.5% of global GDP in the first quarter of 2008 (Twaronite, 2009)), such inefficiencies may lead to significant deadweight loss.

As with land assembly, many of the criteria we defined seem compelling in the corporate acquisitions setting. Shareholders whose holdings are but small parts of diversified portfolios are unlikely to find any non-straightforward system worth the time required for participation. The notion of collective rationality seems particularly compelling, given that it is hard to imagine shareholders making substantial *individual* investments in the value of the firm; most investments are likely to be collective; hence, protecting collective incentives is of the greatest importance. Guaranteeing each individual a share-weighted piece of this collective settlement also seems particularly natural, given that individuals are typically paid their share of the price at which the firm is acquired.

II.C Other applications

To briefly elaborate on some of the other examples discussed in the text:

1. Perhaps the most widely discussed holdout problem in the academic economics literature arises in debt renegotiations and bankruptcy proceedings. Legal rules in many developed countries require the consent of most or even all creditors to a debt renegotiation outside of bankruptcy court and some even require substantial consensus on bankruptcy workouts to avoid liquidation (La Porta et al., 1998). Consent thresholds for workouts differ across countries, but are particularly high in the United States, while lower in Europe and East Asia (Lee, 2007). According to BankruptcyData.com, the assets of United States firms filing for bankruptcy in 2008 was more than a trillion dollars.

2. To avoid problems of collaboration, it is common for an investor to pool together many complementary patented innovations and license them jointly (Lerner and Tirole, 2004). This requires the participation of many complementary patent holders. According to Lerner (2008) more than $100 billion dollars of US output comes from patents held in pools.

References

actionaid, "The Uncertain Domain of Resettlement and Rehabilitiation in India."

Armour, John and David Skeel, "Who Writes the Rules for Hostile Takeovers, and Why? The Particular Divergence of U.S. and U.K. Takeover Regulation," *Georgetown Law Journal*, 2007, *95* (6), 1727–1794.

Bergstrom, Ted, "Cournot Equilibrium in Factor Markets," 1978. http://works.bepress/com/ted_bergstrom/subject_areas.html#Economic Theory.

Berliner, Dana, *Opening the Floodgates: Eminent Domain Abuse in the Post-Kelo World*, Castle Coalition, 2006.
Berman v. Parker

Berman v. Parker*, 1954, 348 U.S. (26).*

Bukart, Mike and Fausto Panuzi*, "Mandatory Bids, Squeeze-out, Sell-out and the Dynamics of the Tender Offer Process," 2003. http://papers.ssrn.com/sol3/papers.cfm?abstract_id=420940.*

Bulow, Jeremy I. and John Roberts*, "The Simple Economics of Optimal Auctions," Journal of Political Economy, 1989, 97 (5), 1060–1090.*

Castle Coalition, *"Legislative Center," 2009.*

Cevallos, Diego, *"Mexico: A Naked Call for Indigenous Peoples' Right to Land," IPS News, January 2nd 2008.*

Cournot, A. Augustin, *Recherches sur les Principes Mathematiques de la Theorie des Richess, Paris, 1838.*

Cripps, Martin W. and Jeroen M. Swinkels, *"Efficiency of Large Double Auctions," Econometrica, 2006, 74 (1), 47–92.*

Croft, Elizabeth and Hank Donker, *"Why Bidders Like Creeping Takeovers," Journal of Corporate Accounting and Finance, 2006, 17 (2), 45–51.*

Dana, David and Thomas W. Merrill, *Property: Takings, New York, NY: Foundation Press, 2002.*

de Vattel, Emerich, *The Law of Nations or the Principles of Natural Law, Washington, DC: Carnegie Institution of WashingtonCarnegie Institution of Washington, 1758.*

Denyer, Lucy, *"Buying Property in Ten of the World's Top Resorts," The Sunday Times, 2008, August 3.*

der Elst, Cristoph Van and Lientje S. F. Van den Steen, *"Opportunities in the M&A Aftermarket: Squeezing Out and Selling Out," 2006. http://papers.ssrn.com/sol3/papers.cfm?abstract_id=933609.*

Ellickson, Robert C., *"Alternatives to Zoning: Covenants, Nuisance Rules, and Fines as Land Use Controls," University of Chicago Law Review, 1973, 40 (4), 683–781.*

Frank, Charles R., *"Entry in a Cournot Market," Review of Economic Studies, 1965, 32 (3), 245–250.*

Grossman, Zachary, Jonathan Pincus, and Perry Shapiro, *"A Second-Best Mechanism for Land Assembly," 2010. Departmental Working Paper, Department of Economics, UC Santa Barbara.*

Grotius, Hugo, *De Jure Belli Ac Pacis Libri Tres, Washington, DC: Carnegie Institution of Washington, 1646.*

Guriev, Sergei, Olga Lazareva, Andrei Rachinsky, and Sergei Tsukhlo, *"Corporate Governance in Russian Industry," Problems of Economic Transition, 2004, 47 (3), 6–83.*
Hawaii Housing Authority v. Midkiff

Hawaii Housing Authority v. Midkiff, *(83–141)*, 1984, *467 U.S.* (229).

Johnson, Stephen and Sara J. Fitzgerald, "The United States and Mexico: Partners in Reform," *Backgrounder*, 2003, *1715.*
Kelo v. City of New London, Connecticut

Kelo v. City of New London, Connecticut, *(04–108), 2005, 545 U.S. (469).*

Kirchmaier, Tom, Jeremy Grant, and Jodie Krishner, *"Financial Tunnelling and the Mandatory Bid Rule," 2009. http://papers.ssrn.com/sol3/papers.cfm?abstract_id=613945.*

Kyle, Albert S., *"Continuous Auctions and Insider Trading," Econometrica, 1985, 53 (6), 1315–1335.*

La Porta, Rafael, Florencio Lopez de Silanes, Andrei Shleifer, and Robert W. Vishny, *"Law and Finance," Journal of Political Economy, 1998, 106 (6), 1113–1155.*

Lee, Hyun Chul, *"Efficient and Inefficient Debt Restructuring: A Comparative Analysis of Voting Rules in Workouts," Cornell International Law Journal, 2007, 40 (3), 661–690.*

Lerner, Josh, *"Monetizing IP: The Executive's Challenge," Working Knowledge, 2008, June (9), http://hbswk.hbs.edu/item/5925.html.*

_ **and Jean Tirole**, *"Efficient Patent Pools," American Economic Review, 2004, 94 (3), 691–711.*

Locke, John, *Two Treatises on Government, London, U.K.: Butler, 1689.*

Mailath, George J. and Andrew Postelwaite, *"Asymmetric Information Bargaining Problems with Many Agents," Review of Economic Studies, 1990, 57 (3), 351–367.*

Menezes, Flavio and Rohan Pitchford, *"A Model of Seller Holdout," Economic Theory, 2004, 24 (2), 231–253.*

Merrill, Thomas W., *"The Economics of Public Use," Cornell Law Review, 1986, 72 (1), 61–116.*

Miceli, Thomas J. and Kathleen Segerson, *"A Bargaining Model of Holdouts and Takings," American Law and Economics Review, 2007, 9 (1), 1–30.*

Michelman, Frank I., *"Property, Utility, and Fairness: Comments on the Ethical Foundations of Just Compensation Law," Harvard Law Review, 1967, 80 (6), 1165–1258.*

Milgrom, Paul, *Putting Auction Theory to Work*, Cambridge, UK: Cambridge University Press, 2004.

Morton, Justin R., "Post-Kelo Legislation on Eminent Domain Impacts Valuations," *Appraisal Journal*, 2006, *76* (4), 383–385.

Myerson, Roger B. and Mark A. Satterthwaite, "Efficient Mechanisms for Bilateral Trading," *Journal of Economic Theory*, 1981, *29* (2), 265–281.

Posner, Richard A., "Foreword: A Political Court," *Harvard Law Review*, 2005, *119* (1), 31–102.

Pufendorf, Samuel, *De Officio Hominis et Civis Juxta Legem Naturalem Libri Duo*, Oxford, UK: Oxford University Press, 1682.

Rich, Jennifer L., "Brazilian Slums Prove Location Reigns Supreme in Real Estate," *New York Times*, 2001, Sunday, December 2.

Rob, Rafael, "Pollution Claim Settlements under Private Information," *Journal of Economic Theory*, 1989, *47* (2), 307–333.

Rustichini, Aldo, Mark A. Satterthwaite, and Steven R. Williams, "Convergence to Efficiency in a Simple Market with Incomplete Information," *Econometrica*, 1994, *62* (5), 1041–1063.

Satterthwaite, Mark A. and Steven R. Williams, "The Rate of Convergence to Efficiency in the Buyer's Bid Double Auction as the Market Becomes Large," *Review of Economic Studies*, 1989, *56* (4), 477–498.

Schmidt, Ronald H. and William C. Gruben, "Ejido Reform and the NAFTA," *FRBSF Weekly Letter*, 1992, *92* (34).

Shavell, Steven, "Eminent Domain versus Government Purchase of Land Given Imperfect Information about Owner's Valuations," 2007. http://lsr.nellco.org/cgi/viewcontent.cgi?article=1386&context=harvard_olin.

Shimizu, Kaho, "Narita to Extend Runway, but Which Way?," *Japan Times*, 2005, April 20.

Sonnenschein, Hugo, "The Dual of Duopoly is Complimentary Monopoly: or, Two of Cournot's Theories are One," *Journal of Political Economy*, 1968, *76* (2), 316–318.

Spengler, Joseph J., *"Vertical Integration and Antitrust Policy," Journal of Political Economy, 1950, 50 (4), 347–352.*

Twaronite, Lisa, *"Corporate Investment-Grade Bond Volume Surges," MarketWatch, 2009, http://www.marketwatch.com/story/corporate-investment-grade-bond-volume-surges?pagenumber=2.*

Van Bynkershoek, Cornelius, *Quaestionum Juris Publici Libri Duo, Oxford, UK: Clarendon, 1737.*

Wolf, Michael Allan, *"Private Property, Community Development, and Eminent Domain," in Robin Paul Malloy, ed., Private Property, Community Development, and Eminent Domain, Burlington, VT: Ashgate, 2008.*